

Tripartite quantum state mapping and discontinuous entanglement transfer in a cavity QED open system

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Received 29 March 2010

Accepted for publication 4 June 2010

Published 30 September 2010

Online at stacks.iop.org/PhysScr/T140/014015

Abstract

We describe the transfer of quantum information and entanglement from three propagating (radiation) to three localized (atomic) qubits via cavity modes resonantly coupled to atoms in the presence of a common reservoir. Upon addressing the full dynamics of the resulting nine-qubit open system, we find that once the cavities are fed, fidelity and transferred entanglement are optimal, while their peak values exponentially decrease due to dissipative processes. The external radiation is then turned off and quantum correlations oscillate between atomic and cavity qubits. For a class of mixtures of W and GHZ input states, we deal with a discontinuous exchange of entanglement among the subsystems, facing the still open problem of entanglement sudden death and birth in a multipartite system.

PACS numbers: 03.67.Mn, 42.50.Pq

1. Introduction

As early as in 1935 Einstein with Podolski, Rosen [1] and Schrödinger [2] drew attention to correlations in quantum composite systems and the problems raised by their properties. Much later, theoretical [3] and experimental [4] cornerstones elucidated the issue of nonlocality. Entanglement is currently viewed as the key resource for quantum information processing [5], where it allowed a number of achievements such as teleportation [6], cryptography [7] and enhanced measurements [8]. The deep meaning of multipartite entanglement, its quantification and detection [9] as well as its possible applications are objects of massive investigation, touching the heart of quantum physics and a number of its branches. In this paper, we study the entanglement dynamics of a multipartite open system in cavity quantum electrodynamics (CQED) [10] by a model that is not so far from physical implementation with the current technology of optical cavities [11]. In particular, we study a paradigmatic example to investigate multipartite entanglement transfer and swapping [12, 13], which are fundamental processes for the implementation of quantum interfaces and memories for quantum networks [14, 15].

In addition, we deal with the recently discovered [16] and observed [17] phenomenon of entanglement sudden death (ESD) (and birth (ESB)), consisting of an abrupt vanishing (and raising) of quantum correlations. The problem is still open with respect to the multipartite entanglement classification, but we can shed some light at least on the ESD and ESB of the fully tripartite entanglement. The interplay among all these aspects in the presence of external environments is investigated.

2. The multipartite open system model

We consider three entangled radiation modes that are injected and resonantly coupled to three separated optical cavities, each of them containing a trapped two-level atom whose transition frequency is resonant with the cavity mode. Each cavity mode is coupled to an external environment by an amplitude damping channel with a decay rate k and each atom can spontaneously emit a photon with a rate γ . The evolution of the whole system density operator $\hat{\rho}(t)$ can be described by a Master Equation (ME) in the Lindblad form that we have solved by means of

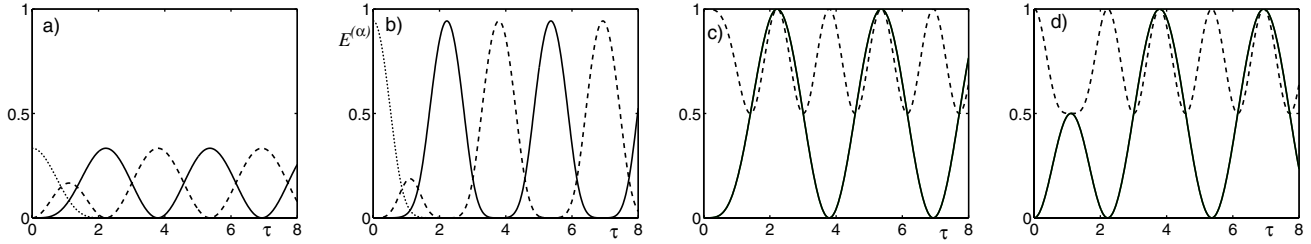


Figure 1. Dynamics for the external field in a W state: (a) mean photon numbers $N^{(c)}$ (dashed), $N^{(f)}$ (dotted) and atomic probability p_c (solid); (b) tripartite negativity $E^{(\alpha)}$ for atoms (solid), cavity modes (dashed) and external field (dotted); (c) atomic fidelity $F^{(a)}$ (solid) and purity $\mu^{(a)}$ (dashed); (d) cavity mode fidelity $F^{(c)}$ (solid) and purity $\mu^{(c)}$ (dashed).

the Monte Carlo wave function method [18]. We identify a set of six collapse operators $\hat{C}_{c,J} = \sqrt{\tilde{k}}\hat{c}_J$ and $\hat{C}_{a,J} = \sqrt{\tilde{\gamma}}\hat{\sigma}_J$ ($J = A, B, C$), where $\tilde{k} = k/g_a$ and $\tilde{\gamma} = \gamma/g_a$ are dimensionless cavity and atomic decay rates scaled to the atom–cavity coupling constant g_a (taken equal for each cavity mode–atom subsystem). The effective Hamiltonian is $\hat{H}_e = \frac{\hat{H}^I}{g_a} - \frac{i\hbar}{2} \sum_J [\hat{C}_{c,J}^\dagger \hat{C}_{c,J} + \hat{C}_{a,J}^\dagger \hat{C}_{a,J}]$, with the system Hamiltonian in the interaction picture $\hat{H}^I = \sum_J [\hbar g_a (\hat{c}_J \hat{\sigma}_J^\dagger + \hat{c}_J^\dagger \hat{\sigma}_J) + i\hbar g_c (\hat{c}_J \hat{f}_J^\dagger - \hat{c}_J^\dagger \hat{f}_J)]$. Here $\hat{c}_J, \hat{c}_J^\dagger$ ($\hat{f}_J, \hat{f}_J^\dagger$) are the quantum harmonic oscillator operators for the cavity (external radiation) modes, $\hat{\sigma}_J, \hat{\sigma}_J^\dagger$ the raising and lowering operators for the atomic qubits and g_c the cavity–input field coupling constant.

2.1. Subsystems dynamics, state mapping and entanglement transfer

We have solved the dynamics for several types of entangled external radiation prepared in mixed or pure states, cavity mode prepared in the vacuum state and different types of atomic preparation. Here we concentrate on the case of atoms in the lower energy state $|ggg\rangle_a$ and the external field prepared in a qubit-like entangled state, because this is the condition for maximum entanglement transfer to the atomic subsystem [13]. Overall, we are thus dealing with a nine-qubit system. From now on, for the sake of simplicity, we take equal coupling constants of the cavity modes with the atoms and the external radiation ($g_c = g_a \equiv g$) and we deal with dimensionless times $\tau \equiv gt$. We first consider the case of negligible cavity and atomic decays. We show that it is possible to map the initial external radiation states onto the atomic and the cavity subsystem states, thereby also transferring the initial quantum correlations. Due to the qubit-like form of the external and cavity mode fields, we quantify the tripartite subsystems entanglement by the tripartite negativity $E^{(\alpha)}(\tau)$ ($\alpha = a, c, f$) [19], defined as the geometric mean of the three bipartite negativities [20]. In figure 1 we illustrate the results for the case of external radiation prepared in the W state $|\Psi(0)\rangle_f = (|001\rangle_f + |010\rangle_f + |100\rangle_f)/\sqrt{3}$. We notice that each subsystem (A, B, C) has the same dynamics that is composed of a transient and an oscillating regime. In the transient, each propagating qubit transfers its excitation to the cavity qubit, which in turn passes it onto the atomic one, as shown in figure 1(a). This is the physical process for the transfer of quantum information. The cavity mode, simultaneously coupled to the external field and the atom (describable as a third harmonic oscillator),

exchanges energy according to a Tavis–Cummings dynamics at an effective angular frequency $g\sqrt{2}$ [10, 21]. During the transient up to time $\tau_{\text{off}} = \pi/\sqrt{2}$, the mean photon number $N^{(c)}(\tau) \equiv \langle \hat{c}^\dagger \hat{c} \rangle(\tau)$ in each cavity completes a cycle. In the same period, the atomic excitation probability $p_c(\tau)$ reaches its maximum value, while the input field has completely entered the cavity, i.e. its mean photon number $N^{(f)}(\tau) \equiv \langle \hat{f}^\dagger \hat{f} \rangle(\tau)$ vanishes.

In figure 1(b) we show $E^{(\alpha)}$ and see that up to $\tau_{\text{off}}/2$ the entanglement of the external driving field is mainly transferred to the cavity modes and then allows the build-up of atomic entanglement. We note that $E^{(a)}(\tau_{\text{off}}) \cong 0.94$, which is the value of the tripartite negativity for the injected W state. For the time evolution of the atomic and cavity mode subsystems described by the density operator $\hat{\rho}_\alpha(\tau)$ ($\alpha = a, c$), we show in figures 1(c) and (d) the purity $\mu^{(\alpha)}(\tau) = \text{Tr}_\alpha[\hat{\rho}_\alpha^2(\tau)]$ and the fidelity $F^{(\alpha)}(\tau) = {}_\alpha \langle \Psi(0) | \hat{\rho}_\alpha(\tau) | \Psi(0) \rangle_\alpha$ with respect to a pure state $|\Psi(0)\rangle_\alpha$ that is a map of the initial external field $|\Psi(0)\rangle_f$, with the correspondence $|0\rangle_f \leftrightarrow |g\rangle_a (|0\rangle_c)$ and $|1\rangle_f \leftrightarrow |e\rangle_a (|1\rangle_c)$ for the atoms (cavity mode) states. We see that up to τ_{off} , the dynamics maps the whole initial state $|\Psi(0)\rangle_f \otimes |000\rangle_c \otimes |ggg\rangle_a$ onto $|000\rangle_f \otimes |000\rangle_c \otimes \hat{U}_{-\pi/2} |\Psi(0)\rangle_a$, where $\hat{U}_\phi^{(a)} = \otimes_J e^{-i\phi \hat{\sigma}_J^\dagger \hat{\sigma}_J}$ is a local phase rotation, that in the case of W states simply acts as a global phase factor $+i$. Note that the maximum of $E^{(c)}(\tau_{\text{off}}/2)$ does not correspond to a pure state, i.e. the initial state $|\Psi(0)\rangle_f$ cannot be mapped onto the cavity modes during the transient regime.

At the end of the transient regime the external radiation is turned off and the subsequent dynamics can be described by a triple Jaynes–Cummings [22] model ruled by oscillations at the vacuum Rabi frequency $2g$, hence with a dimensionless period π as shown by the cavity mean photon number and atomic probability in figure 1(a). The purities $\mu^{(a,c)}(\tau)$ in figures 1(c) and (d) oscillate at a double frequency between pure full tripartite entangled and separable states. In particular, at times $\tau_m = \tau_{\text{off}} + m\pi$ ($m = 0, 1, 2, \dots$) the atoms are in the entangled states $\hat{U}_\phi^{(a)} |\Psi(0)\rangle_a$, where $\phi = \mp \frac{\pi}{2}$ and $-(+)$ applies in correspondence to even (odd) values of m , that are the peaks of $E^{(a)}(\tau)$ in figure 1(b). At times $\tau_n = \tau_{\text{off}} + (n + \frac{1}{2})\pi$ ($n = 0, 1, 2, \dots$) the cavity modes are in the qubit-like state $\hat{U}_\phi^{(c)} |\Psi(0)\rangle_c$, where $\hat{U}_\phi^{(c)} = \otimes_J e^{-i\phi \hat{c}_J^\dagger \hat{c}_J}$ is a local rotation such that $\phi = 0$ ($\phi = \pi$) for even (odd) values of n . As regards bipartite subsystem entanglement, evaluated after tracing over all the other qubits, we find sizeable two-atom and two-cavity entanglement (negativity peaks $\simeq 0.41$), as well as some entanglement for all the other qubit pairs (equal negativity peaks $\simeq 0.08$).

Table 1. Effect of cavity decay rate \tilde{k} on $F^{(a)}(\tau_m)$ at the first peak ($m = 0$) and $E^{(a)}(\tau_m)$ at the first two peaks ($m = 0, 1$), and on $F^{(c)}(\tau_n)$ ($n = 0$) and $E^{(c)}(\tau_n)$ ($n = 0, 1$) for injected field states W and GHZ.

	$F^{(a)}(\tau_0)$	$E^{(a)}(\tau_0)$	$E^{(a)}(\tau_1)$	$F^{(c)}(\tau_0)$	$E^{(c)}(\tau_0)$	$E^{(c)}(\tau_1)$
β_W	0.55	1.09	4.59	1.41	3.22	6.47
β_{GHZ}	0.77	1.10	4.69	1.88	2.80	6.11

Starting, instead, from an external field in the qubit-like GHZ state $|\Psi(0)\rangle_f = \frac{1}{\sqrt{2}}(|000\rangle_f + |111\rangle_f)$, we find a quite similar time evolution but different values for the maxima of the quantities evaluated in figures 1(a) and (b). In particular, at times τ_m we have the maximum entanglement transfer with $E^{(a)} = 1$, that is again the same value of $|\Psi(0)\rangle_f$, and the atomic states are mapped onto $(|000\rangle_a \mp i|111\rangle_a)/\sqrt{2}$. On the other hand, at times τ_n the cavity modes are in the state $(|000\rangle_c \pm |111\rangle_c)/\sqrt{2}$. The two-qubit entanglement is null for any pair, reflecting the lack of subsystem entanglement in a GHZ state, except for the three directly coupled atom–cavity pairs, where the negativity peaks occur at about 0.21 at any quarter and three quarters of a period. We remark that the above state mapping can also be obtained for any qubit-like state $|\Psi(0)\rangle_f$ written in a generalized Schmidt decomposition [23].

2.2. Dissipative effects

From the perspective of experimental implementation of our scheme for quantum information purposes, an important issue is the effect of dissipation on both state mapping and entanglement transfer. For cavity decay rates in the range $0 < \tilde{k} < 0.4$, we calculated the fidelities $F^{(\alpha)}(\tau)$ at the first peaks (after the transient) as well as the tripartite negativities at the first two peaks $E^{(\alpha)}$ of both atomic and cavity field subsystems, for external field preparations in states W and GHZ. The behaviour of all these quantities can be well described as $f(\tilde{k}) = f(0)e^{-\beta\tilde{k}}$, where the values of decay rates β are reported in table 1. As expected, quantum state mapping and entanglement transfer are far more efficient onto atomic than cavity qubits. Actually, these processes involve the atoms when the cavity modes, coupled to the external environment, are negligibly excited. Cavity modes are instead affected by dissipation already in the transient regime, which leads to a reduction of the relevant peaks as well as asymmetries in their profile (decay slower than build-up). All these features, imprinted in the transient, become quite manifest in the subsequent cavity dynamics.

We also analysed the effect of the spontaneous emission of atomic excited levels on state mapping and entanglement transfer processes. We consider, as an example, the case of decay rate $\tilde{k} = 0.1$ and values of dimensionless parameter $\tilde{\gamma}$ up to 0.1. The exponential approximation is less accurate, and we report in table 2 the results for exponential decay rate δ only at the first peaks of fidelity and tripartite negativity for both atomic and cavity mode subsystems.

3. Fully tripartite ESD and ESB

Let us now consider the class of mixed qubit-like states for the injected field $\hat{\rho}_f(0) = p|GHZ\rangle\langle GHZ| + (1-p)|W\rangle\langle W|$,

Table 2. Effect of atomic decay rate $\tilde{\gamma}$ on $F^{(a)}(\tau_m)$ and $E^{(a)}(\tau_m)$ at the first peak ($m = 0$), and on $F^{(c)}(\tau_n)$ and $E^{(c)}(\tau_n)$ for $n = 0$, for injected field states W and GHZ.

	$F^{(a)}(\tau_0)$	$E^{(a)}(\tau_0)$	$F^{(c)}(\tau_0)$	$E^{(c)}(\tau_0)$
δ_W	1.01	2.06	1.85	4.07
δ_{GHZ}	1.57	2.21	3.14	4.51

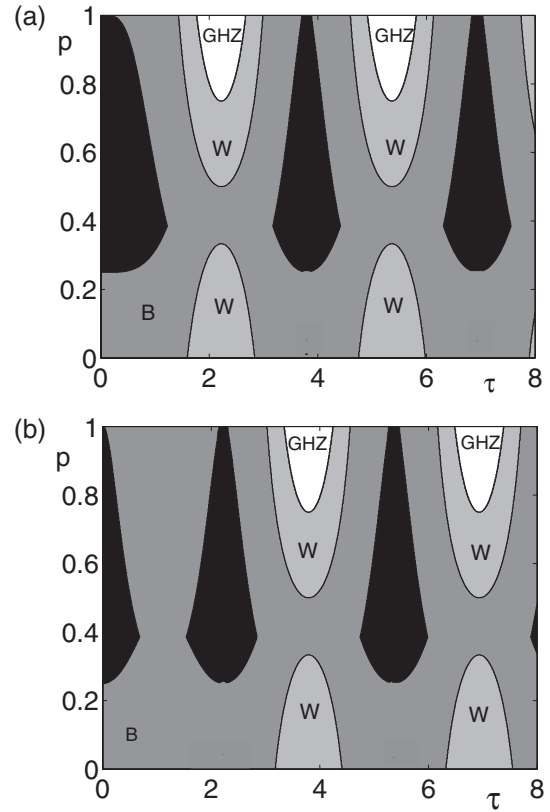


Figure 2. Entanglement classification in the parameter space $\{p, \tau\}$ for an external field prepared in the mixed GHZ and W state $\hat{\rho}_f(0)$. (a) atomic subsystem, (b) cavity mode subsystem.

$0 \leq p \leq 1$. By analytical and numerical results, we show that our scheme for entanglement transfer and swapping is also relevant for the observation of sudden disentanglement and entanglement effects. Since the tripartite negativity $E^{(\alpha)}(\tau)$ ($\alpha = a, c$) is an entanglement measure that provides only a sufficient condition for entanglement detection, we cannot properly talk about ESD or ESB for these kinds of states in the whole parameter space $\{p, \tau\}$. Nevertheless, we can classify the atomic state by using the entanglement witnesses [24] $\hat{W}_G = \frac{3}{4}\hat{I} - |GHZ\rangle\langle GHZ|$, $\hat{W}_{W2} = \frac{1}{2}\hat{I} - |GHZ\rangle\langle GHZ|$ and $\hat{W}_{W1} = \frac{2}{3}\hat{I} - |W\rangle\langle W|$, and analyse the discontinuous evolution of entanglement focusing only on the fully tripartite entanglement properties. For negligible dissipation and at times τ_m , the initial state of radiation is mapped onto the atoms, and then the entanglement classification is known. The state is of class GHZ for $\frac{3}{4} \leq p \leq 1$, of class W for $0 \leq p < \frac{1}{3}$ and $\frac{1}{2} \leq p < \frac{3}{4}$, and biseparable (B) for $\frac{1}{3} \leq p < \frac{1}{2}$. Outside times τ_m , we can still make a partial entanglement classification that is shown in figure 2(a) (figure 2(b)) for atomic (cavity field) states. Since the tripartite negativity is zero in the black regions of figure 2, we cannot exclude the presence of bound entangled states. With

this knowledge on the entanglement properties in parameter space, we can affirm that there is a discontinuity for the full tripartite entanglement. Fixing, for instance, a value of $p < 0.25$ and looking at the time evolution of the atomic state, we notice that it suddenly acquires a fully tripartite entanglement entering the W region and loses this property after some finite time exiting that region. This effect can be addressed as an ESD and ESB of the fully tripartite inseparability only. Moreover, since the systems share among the subsystems energy and entanglement in a periodic way, we can also highlight that discontinuities in the fully tripartite entanglement are exchanged among them. This occurs after the transient, where the cavity modes do not exhibit genuine tripartite entanglement (see figure 2(b)) but only support the transfer of quantum correlations from the input field to the atoms.

4. Conclusions

In this paper, we have addressed the transfer of quantum information and entanglement from three propagating qubits to three localized ones, focusing on the basic physical features characterizing multiqubit state mapping in a CQED setting. We analysed the effect of dissipation at the times when the transfer protocol is optimal, considering the atoms and the cavities both in contact with the same reservoir at zero temperature. We also derived the conditions for the repeated occurrence of discontinuous exchange of quantum correlations among the multipartite subsystems. Our scheme could be implemented by combining current advances with optical fibres, cavities and trapped atoms [11, 25, 26].

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