International Journal of Quantum Information
Vol. 9, Suppl. (2011) 27–38

DOI: 10.1142/S0219749911007009

DECOHERENCE OF GAUSSIAN AND NONGAUSSIAN PHOTON-NUMBER ENTANGLED STATES IN A NOISY CHANNEL

MICHELE ALLEGRA

Dipartimento di Fisica Teorica, Università degli Studi di Torino, I-10123 Torino, Italy

PAOLO GIORDA

I.S.I. Foundation I-10133 Torino, Italy

MATTEO G. A. PARIS

Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy

Received 24 August 2010

We consider photon-number entangled states (PNES) and study the degradation of their entanglement in a noisy channel, using different separability criteria and a recently proposed measure of nonGaussianity as key tools. Upon comparing Gaussian and nonGaussian states within the class, we collect some evidence that Gaussian states are maximally robust against noise, i.e. the complete loss of entanglement occurs in maximal time. However, the gap with respect to nonGaussian states is negligible for sufficiently high energy of the states.

Keywords: NonGaussianity; entanglement; noisy quantum channel.

1. Introduction

Many quantum communication protocols are based on the exploitation of entanglement between two distant physical systems.[2] Entangled correlations are usually created by a local process, so that bringing the systems far apart involves transmission of either or both through a noisy channel. During transmission, the noise causes a continuous degradation of entanglement, which may eventually vanish. The successful accomplishment of the protocols then requires ability to control the effect of noise on entangled resources.

Several cases of noisy dynamics are described in the literature, covering both discrete and continuous variable (CV) systems. However, most studies dealing with CV systems have so far been restricted to Gaussian resources. This should not come as a surprise: indeed CV quantum information arose and was developed with Gaussian states,[3–7] which meet the two fundamental requirements of theoretical simplicity (a complete description of the states is achieved by taking into account only first- and second-order moments) and experimental feasibility (Gaussian states are created and manipulated through Gaussian operations, which correspond to linear optical devices). Moreover, most communication protocols originally devised for finite-dimensional Hilbert spaces admit natural analogues in terms of CV Gaussian states. Nevertheless, recent developments suggest that going beyond Gaussian states and operations may allow for considerable progress in long-distance quantum communication: it has been shown, for instance, that nonGaussianity plays an important role in enhancing entanglement distillation[8–12] and swapping, quantum memories,[13] cloning[14] and teleportation.[15–17] For this reason, decoherence of nonGaussian entangled resources deserves consideration as well.[18]

A recent work of ours[1] was centered on a basic question which arises quite naturally in this context: *in the case of transmission throuh a noisy environment, are nonGaussian states more or less robust than Gaussian, i.e. do they lose entanglement in a longer or shorter time*?

We addressed this question by focusing on a particular case of entangled resource, namely Photon-Number Entangled States (PNES), a class of two-mode CV entangled states comprising both Gaussian and nonGaussian states, and on a particular instance of noisy environment, namely a Gaussian channel governed by a Markovian Master Equation. The tools commonly used to analyse Gaussian entanglement (e.g. Simon's separability criterion) are clearly insufficient to investigate the degradation of nonGaussian PNES' entanglement. Consequently, we had to resort to additional tools, including several separability criteria and a recently introduced measure of nonGaussianity. As a result, we were able to compare the robustness of Gaussian and nonGaussian entanglement.

This paper is a detailed review and an extension of that work[1]: we take into account more kinds of noise and present a wider range of results on nonGaussian entanglement evolution. It is organised as follows: in Sec. 2 we introduce PNES and their properties; in Sec. 3 the main features of the noisy channel are explained; in Sec. 4 we briefly review some separability criteria; in Sec. 5 our main results are described; Sec. 6, finally, closes the paper with some concluding remarks.

2. PNES

As stated in the introduction, we seek for a way to compare the degradation of Gaussian and nonGaussian entanglement in a noisy channel. A possible strategy is to focus on a particular class of states. We have chosen to work with a broad and meaningful class of CV bipartite states endowed with perfect correlations in the number of photons: Photon-Number Entangled States (PNES). These are states $|\psi\rangle$

having Schmidt decomposition in the Fock basis, i.e.

$$|\psi\rangle = \sum_{n=0}^{\infty} \psi_n |n\rangle|n\rangle. \tag{1}$$

For the sake of simplicity, we shall consider real positive coefficients $\psi_n \in \mathbb{R}$, $\psi_n > 0$, $\sum_{n=0}^{\infty} \psi_n^2 = 1$.

The motivation for the choice of PNES is manifold. First, PNES are sufficiently simple but at the same time meaningful: several experimental realizations of PNES have been reported[19–22] and many PNES-based quantum communication schemes have been proposed.[23–27] Second, the set of PNES is particularly suitable for our purpose since it contains mostly nonGaussian states but includes, as a subclass, two-mode squeezed vacua, i.e. the basic Gaussian resource for CV quantum information, thus allowing for a direct comparison between Gaussian and nonGaussian states. Finally, PNES are good candidates for the role of entangled resource in long-distance quantum communication, because their entanglement is robust against phase diffusion noise and losses (see Sec. 5).

Numerical calculations require specific numerical choices of the coefficients ψ_n. We shall therefore consider several special subclasses of PNES with specific parametric dependence (we omit normalization):

(i) the two-mode squeezed vacua or twin-beam states (TWB) $\psi_n \propto x^n$ with $0 \le x < 1$. These are the sole Gaussian states within the PNES class and represent the preferred (Gaussian) resources in protocols based on CV entanglement;
(ii) the photon subtracted two-mode squeezed vacua (PSSV)[31] $\psi_n \propto (n+1)x^{n+1}$ and the photon-added two-mode squeezed vacua (PASV)[32] $\psi_n \propto n x^{n-1}$, which are obtained from the TWB by the experimentally feasible[33–35] operations of photon subtraction $\varrho \to a_1 a_2 \varrho a_1^\dagger a_2^\dagger$ and addition $\varrho \to a_1^\dagger a_2^\dagger \varrho a_1 a_2$ respectively;
(iii) the pair-coherent or two-mode coherently correlated states (TMC)[36,37] with Poissonian profile $\psi_n \propto \lambda^n/n!$, $\lambda \in \mathbb{R}$.

The mean energy of PNES, $E = \langle\psi|a_1^\dagger a_1 + a_2^\dagger a_2|\psi\rangle = 2N$ is given in terms of the mean photon number $N = \sum_{n=0}^{\infty} |\psi_n|^2 n$, whereas correlations between the modes can be quantified by $C = \mathrm{Re} \sum_{n=0}^{\infty} \psi_n^* \psi_{n+1}(n+1)$. In turn, the covariance matrix of a PNES equals that of a symmetric Gaussian state in standard form, with diagonal elements equal to $N + (1/2)$ and off-diagonal blocks given by $\boldsymbol{C} = \mathrm{diag}(C, -C)$. The entanglement of PNES can be assessed by the von Neumann entropy of the partial traces, $\epsilon_0 = \sum_n |\psi_n|^2 \log |\psi_n|^2$.

For numerical purposes, states have to be truncated: i.e. one deals with $|\psi\rangle = \sum_{n=0}^{D} \psi_n |n\rangle|n\rangle$ where D is a suitable truncation threshold. We shall always consider $D = 20$ and energies in the range $0 \le E \le 10$. The adopted truncation results in a negligible error for all subclasses of states.

3. Noisy Environment

If a system is coupled to its environment, the Hamiltonian dynamics appropriate for an isolated system $\varrho(t) = e^{-iHt}\varrho(0)e^{iHt}$ is to be replaced by a class of linear maps Λ_t: $\varrho(t) = \Lambda_t\varrho(0)$ which must be completely positive and trace-preserving in order for Λ_t to be physical. The environment is usually assumed to be Markovian, which is formalized by the semigroup property $\Lambda_{t_1+t_2} = \Lambda_{t_1}\Lambda_{t_2}$. Lindblad has shown[38] that $\Lambda_t = e^{\mathcal{L}t}$ is a (quantum) semigroup iff $\mathcal{L}\varrho = -i[H,\varrho] + \sum_{j\in I}[V_j\varrho, V_j^*] + [V_j, \varrho V_j^*]$ where H is Hermitian, which implies that the evolution of ϱ is governed by the Master Equation $d/dt\varrho = \mathcal{L}\varrho = -i[H,\varrho] + \sum_j [V_j\varrho, V_j^\dagger] + [V_j, \varrho V_j^\dagger]$.

In our case, the propagation in noisy channels can be modeled as the interaction of the two modes with two independent thermal baths of oscillators. In the Markovian approximation, the resulting dynamics is a Gaussian channel, governed by the two−mode Lindblad-type Master Equation[39]

$$\dot{\varrho} = \sum_{j=1,2} \frac{\Gamma}{2} N_j L[a_j^\dagger]\varrho + \frac{\Gamma}{2}(N_j+1)L[a_j]\varrho \tag{2}$$

describing losses and thermal hopping in the presence of (local) non−classical fluctuations of the environment. Dot stands for time−derivative and the Lindblad superoperator is defined by $L[O]\varrho \equiv 2O\varrho O^\dagger - O^\dagger O\varrho - \varrho O^\dagger O$. Γ is a loss coefficient and N_j are the mean photon-numbers in the stationary state, which is a two-mode thermal state $\nu_{12} = \nu_1 \otimes \nu_2$, $\nu_j = (1/(N_j+1)) \sum_{n=0}^{\infty} (N_j/(N_j+1))^n |n\rangle_{jj}\langle n|$. We consider baths at equal temperature $N_1 = N_2 = N_T$.

As shown by D'Ariano,[40] the above Master Equation admits the operator solution:

$$\varrho(t) = \Lambda_t\varrho(0) = \mathrm{Tr}_{34}[U_t(\varrho(0)\otimes\nu_{34})U_t]. \tag{3}$$

In the formula above, Λ_t denotes the evolution map corresponding to the noisy channel; 3, 4 are two additional fictitious modes in a thermal state $\nu_{34} = \nu_3 \otimes \nu_4$ which mirrors the asymptotic thermal state ν_{12} of the system; $U_t = U_{13}(\zeta_t) \otimes U_{24}(\zeta_t)$; $U_{ij}(\zeta_t) = \exp(\zeta_t a_i^\dagger a_j - \zeta_t^* a_j^\dagger a_i)$ is the two-mode mixing operator, with $\zeta_t = \arctan{(e^{\Gamma t}-1)^{1/2}}$.

In addition to losses and thermal hopping, one can consider phase diffusion noise, modeled by a ME[41]

$$\dot{\varrho} = \sum_{j=1,2} \frac{\Gamma}{2} L[a_j^\dagger a]\varrho. \tag{4}$$

The solution of Eq. (4) for initial PNES, $\varrho = \sum_{mn}^{\infty} \psi_n\psi_m |nn\rangle\langle mm|$, is simply

$$\varrho(t) = \sum_{mn}^{\infty} \psi_n\psi_m e^{-\Gamma t|m-n|^2} |nn\rangle\langle mm|. \tag{5}$$

4. Separability Criteria and NonGaussianity

In order to control the evolution of entanglement of PNES under the action of noise, we ought to make use of several tools, including many entanglement criteria and a recently introduced measure of nonGaussianity.

As it is well-known, in the CV case a necessary-and-sufficient separability criterion exists only for Gaussian states: it is Simon's criterion (SI).[42] The latter is equivalent to the positivity of the partial transpose density matrix and can be cast in the following form[43]: a Gaussian state is separable iff $\tilde{d}_- < 1/2$, where $\tilde{d}_-$ is the least symplectic eigenvalue of the covariance matrix corresponding to the partial-transposed state. Yet when dealing with nonGaussian states Simon's criterion (which is then equivalent to the separability of a Gaussian state having the same covariance matrix as the given state) is only sufficient for entanglement.

This actually holds for any available criterion: if the state under consideration is entangled, a given test may or may not detect its entanglement; in turn, if a given test does not detect entanglement, we cannot conclude that the state is separable. Consequently, the only way to draw reliable conclusions about the entanglement/separability of a given state is to use several different criteria, which provide independent separability conditions, thus minimizing the possibility of failure in entanglement detection. From the most relevant criteria available for CV systems, we select some which we deem particularly suitable for our case.

First, we consider Shchukin and Vogel's criterion (SH),[44,45] conceived as an extension of the PPT criterion to the nonGaussian case. SH is based on the evaluation of a series of $M \times M$ matrices whose entries are moments up to a given order: non-positive-definiteness of any finite submatrix is a sufficient condition for entanglement. Upon considering first and second-order moments only ($M = 5$) we obtain a condition which is equivalent to Simon's criterion. If one considers larger minors, moments of higher order are involved and we get a stronger condition. We have considered moments up to order 8.

Another class of criteria is based on linear entanglement witnesses (EW).[46] A witness is any operator W such that $\mathrm{Tr}[\varrho_{sep} W] \geq 0$ on any separable state ϱ_{sep}. Then $\mathrm{Tr}[\varrho_{ent} W] < 0$ implies that the state ϱ_{ent} is entangled. Sperling and Vogel (SP)[47] showed that this condition can be rephrased in the following form: a state ϱ is entangled if $\langle\phi|\varrho|\phi\rangle > \max_n\{|m_n|^2\}$ where $|\phi\rangle$ is a pure entangled state with Schmidt coefficients $\{m_n\}$. In our study we test this condition by using 10^4 randomly generated witnesses of the form: $|\phi\rangle = \sum_{n=1}^{D} \phi_n |n\rangle|n\rangle$ with $D = 20$, i.e. the witnesses are themselves truncated PNES. This particular form was chosen since the bath does not create quantum correlations but only destroys those originally present.

The realignment criterion (RE)[48,49] is the last criterion considered. It is based on positivity of a linear contraction map: a state ϱ is entangled if $\|\tilde{\varrho}\| > 1$ where $\|A\|$ denotes the trace norm of operator A and $\langle m|\langle\mu|\tilde{\varrho}|n\rangle|\nu\rangle = \langle m|\langle n|\varrho|\nu\rangle|\mu\rangle$.

The propagation in noisy channels, besides entanglement, also destroys the nonGaussian character of the initial state, which unavoidably evolves towards the asymptotic, Gaussian thermal state. We wish to take into account both processes (separation and Gaussification) in parallel and explore the relations between them. The nonGaussian character of a state ϱ is measured by $\delta(\varrho) = S(\tau) - S(\varrho)$ i.e. the relative entropy between ϱ and the reference Gaussian state τ having the same covariance matrix. Genoni *et al.*[50] showed that $\delta(\varrho)$ is a good measure of non-Gaussianity, i.e. it is non-negative, continuous, invariant under unitary Gaussian maps and nonincreasing under general Gaussian maps.

5. Robustness of PNES Entanglement Against Noise

5.1. *Robustness against phase diffusion noise*

Having gathered all necessary tools, we can now consider the effect of the noisy environment on PNES.

First of all, we shall assess the effect of phase diffusion noise (4). We shall prove that *all Gaussian and nonGaussian PNES are robust against this kind of noise*: indeed by explicitly constructing an EW we can show that phase noise never leads to a separable state, i.e. the entanglement is never destroyed for any value of Γt.[41] An entanglement witness can be constructed[28–30] as $W = (|\epsilon\rangle\langle\epsilon|)^{PT}$, PT denoting partial transposition and $|\epsilon\rangle$ being the eigenvector of ϱ^{PT} corresponding to the least eigenvalue. From (5) we have the partial transpose $\varrho^{PT}(t) = \sum_{mn}^{\infty} \psi_n\psi_m e^{-\Gamma t|m-n|^2}$ $|mn\rangle\langle nm|$. The eigenvalue equation $\varrho^{PT}(t)|\phi\rangle = \lambda|\phi\rangle$ is solved by $\lambda_{nm} = \pm\psi_n\psi_m e^{-\Gamma t|m-n|^2}$, $|\phi^{\pm}_{nm}\rangle = (1/\sqrt{2})(|nm\rangle \pm |mn\rangle)$. The eigenvector corresponding to the least eigenvalue is $|\phi_{01}\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$ and we have $\mathrm{Tr}[\varrho|\phi_{01}\rangle\langle\phi_{01}|] = -\psi_0\psi_1 e^{-\Gamma t} < 0$. This implies the thesis: entanglement is never destroyed for any value of Γt.

5.2. *Robustness against loss and hopping: Simon's criterion*

We now address the effect of losses and thermal hopping. We emphasize that the map Λ_t (3), being the product of two local maps, can never build but only disrupt quantum correlations: as shown below, for any $N_T \neq 0$ we have a complete loss of entanglement within a finite, state dependent, time $t_S = t_S(\varrho)$ which we refer to as the separation time.

Using separability criteria, we obtain lower bounds on separation times. Indeed, for any given criterion K and state ϱ, let us denote by $t_K(\varrho)$ the maximum time for which K proves that ϱ is entangled: clearly $t_K(\varrho)$ is a lower bound for t_S. Upon considering the best bound we have $t_S(\varrho) \geq \max_K t_K(\varrho)$.

The Simon separation time t_{SI} can be computed analytically. At the level of covariance matrix, the map Λ_t induces the evolution $\sigma_t = \sigma_0 e^{-\Gamma t} + \sigma_\infty(1 - e^{-\Gamma t})$ where $\sigma_\infty = \mathrm{diag}(N_T + 1/2, \ldots, N_T + 1/2)$ is the asymptotic thermal state's covariance matrix. The covariance matrix corresponding to the partial-transposed state is given by $\Lambda\sigma_t\Lambda$, where $\Lambda = \mathrm{diag}(1, 1, 1, -1)$,[42] and its least symplectic eigenvalue is $\tilde{d}_- = (N_T + 1/2)(1 - e^{-\Gamma t}) + (N + 1/2)e^{-\Gamma t} - |C|e^{-\Gamma t}$. Therefore, for $N_T \neq 0$ we have a lower bound to separability

$$t_{SI} = \frac{1}{\Gamma}\log\left(1 + \frac{|C| - N}{N_T}\right) \tag{6}$$

We deduce that *if no hopping is present* ($N_T = 0$), *then entanglement of PNES is never destroyed* for any finite value of Γt) since $t_S \geq t_{SI} = \infty$. For ($N_T \neq 0$), however, entanglement is expected to vanish within a finite time.

Let us focus our attention on the dependence of the Simon separation time t_{SI} on the initial nonGaussianity δ_0. We have $\delta_0 = 2f(d_-)$ where $d_- = \sqrt{(N + 1/2)^2 - |C|^2}$ is the least symplectic eigenvalue of the covariance matrix and $f(x) = (x + 1/2) \times \log(x + 1/2) - (x - 1/2)\log(x - 1/2)$ monotonically increases with x.[6,7] Upon defining $g = f^{-1}$, t_{SI} can be written as

$$t_{SI} = \frac{1}{\Gamma}\log\left(1 + \frac{\sqrt{(N + 1/2)^2 - g^2(\delta_0/2)} - N}{N_T}\right) \tag{7}$$

which shows that t_{SI} is a decreasing function of δ_0 at any fixed N, and it is maximized by TWB for which $\delta_0 = 0$.

5.3. *Robustness against loss and hopping: beyond Simon's criterion*

Through Simon's criterion, we have derived an analytical estimate of the PNES separation times, suggesting that they decrease with the initial nonGaussianity and are maximal for Gaussian states, at any fixed energy. However, the Simon separation time is only a lower bound $t_{SI} \leq t_S$. *Can we consider t_{SI} as a reliable estimate of t_S?*

We shall now address this question. By means of separabililty criteria other than Simon's, we shall verify whether we can obtain better bounds on t_S; by means of the nonGaussianity, we shall get an independent estimate on how reliable Simon's criterion is, since when the nonGaussianity is low (i.e. when the states are nearly Gaussian), Simon's criterion is expected to be very reliable.

In order to perform entanglement tests and to compute the nonGaussianity, we must work at the density matrix level, not at the covariance matrix level. Calculations cannot be carried over analytically and we ought to use numerical methods. Using solution (3), the evolved density matrix $\varrho(t)$ can be computed numerically from the initial state $\varrho(0)$ upon truncating the Hilbert space dimension D. We then consider states with maximal photon number $D = 20$ and total energy $0 \leq 2N \leq 10$ (in this range of energies, and for all subclasses of states, the effect of truncation is negligible).

In order to explore the effect of noise in a wide range of conditions and initial states we then consider TMC, PSSV and PASV of different energies. We compute the evolved density matrix for $0 \leq t \leq 15$ in units of inverse loss $1/\Gamma$. At any time t, entanglement is tested with all the above mentioned criteria and the value of the nonGaussianity δ is computed. From these data we evaluate t_K, i.e. lower bounds to separation times according to different criteria, and Gaussification times t_G, i.e. times

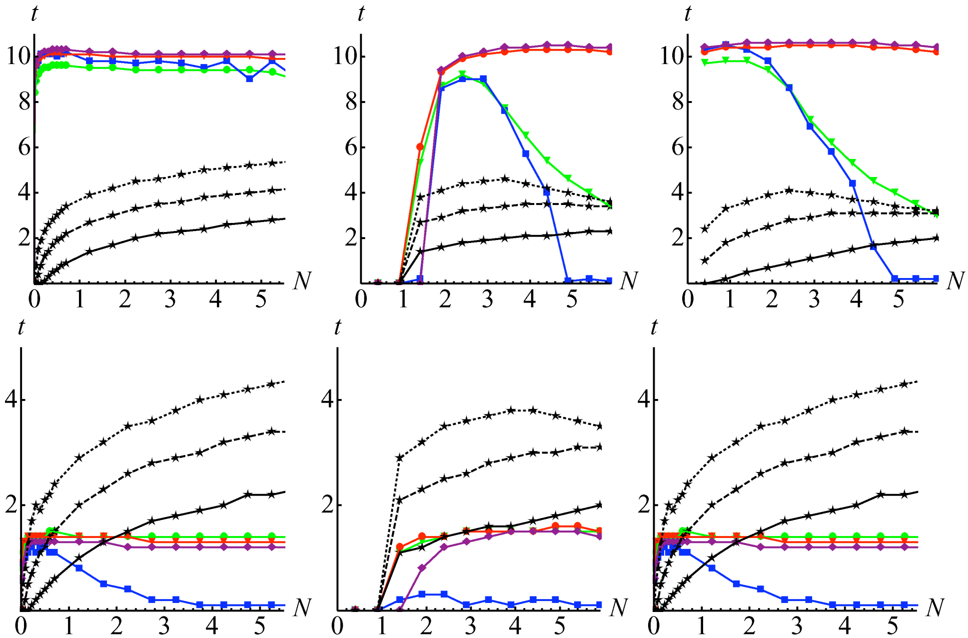

Fig. 1. (Color on line) Separation and Gaussification times for TMC (left), PASV (center), PSSV (right) as a function of the mean energy for low ($N_T = 10^{-5}$, top) and high ($N_T = 10^{-1}$, bottom) temperature. In all plots we report separation times according to different criteria: t_{RE} (green, triangle), t_{SP} (blue, square), t_{SH} (red, circle), t_{SI} (purple, rhombus), and Gaussification times for different thresholds: $\delta_G = 10^{-1}$ (solid black, star), $\delta_G = 10^{-2}$ (dashed black, star), $\delta_G = 10^{-3}$ (dotted black, star).

for which nonGaussianity δ falls below a fixed Gaussification threshold δ_G (we consider different thresholds $\delta_G = 0.1, 0.01, 0.001$). The procedure is then repeated for different values of the temperature T corresponding to N_T in the range $[10^{-5}, 10^{-1}]$.

It turns out that all states exhibit approximately the same qualitative behaviour. In Fig. 1 we report t_K for different PNES subclasses and different criteria as a function of N for the lowest (highest) temperature considered $N_T = 10^{-5}(10^{-1})$. We notice the following facts:

(i) at both temperatures SI and SH criteria yield very similar curves. Also RE shows very good agreement with SI and SH (except for PASV and PSSV at low T). As for the SP criterion, it yields much worse bounds for the separation time. Thus we have numerically proved that for the whole class of states we have considered, the SI, SH and RE criteria yield qualitatively the same results and SI, which offers analytical advantages, is the optimal one.

(ii) both at high and low T, t_K rapidly increases to an asymptotic value $\bar{t}_K$ which is reached at $N \sim 1$; $\bar{t}_K$ is a decreasing function of T, i.e. entanglement loss is strongly affected by the increase of the temperature.

In Fig. 1 we also show Gaussification times as a function of energy. Upon comparing separation and Gaussification times we notice that:

(iii) the behaviour of nonGaussianity is only weakly affected by the increase of T.

(iv) at low T states become nearly Gaussian well before they become separable: $t_G < t_K \leq t_S$; at high T, on the contrary, Gaussification times are greater than our bounds on separation times: $t_G > t_K$.

Observation (iv) deserves futher consideration. Indeed, the relation $t_G < t_K \leq t_S$ implies that for low T the bounds provided by Simon's criterion properly estimate the actual PNES separation times i.e., $t_{SI} \simeq t_S$. This can be understood by first noticing that when $t > t_G$, the states are nearly Gaussian and therefore Simon's criterion is expected to be very reliable. Furthermore, at any $t \geq t_{SI} > t_G$ the reference Gaussian state is obviously separable and thus the nonGaussianity can be compared with a measure of entanglement: the relative entropy[51] $E(\varrho) = \min_{\sigma\in\Omega}[S(\varrho||\sigma)]$ that quantifies the distance between ϱ and the whole set of separable states Ω. When $t \geq t_{SI}$ one has that $E(\varrho) \leq \delta(\varrho) \ll 1$, and this confirms that in this limit the states are very poorly entangled (if they are) and SI allows us to reliably estimate t_S. The fact that other criteria provide very close bounds on t_S strengthens this conclusion. For high T, Gaussification times are greater than t_K provided by all the criteria and we cannot draw the same conclusion, i.e. times t_K must be considered just as lower bounds. However, the relative agreement between different criteria is still an indication that t_K should be regarded as rather effective bounds.

5.4. *Robustness against loss and hopping: extremality of Gaussian entanglement*

From the above considerations, it follows that in the low T regime (where $t_{SI} \simeq t_S$) Gaussian states have maximal separation times at any fixed energy. This is shown in the left panel of Fig. 2 where the separation times and the initial

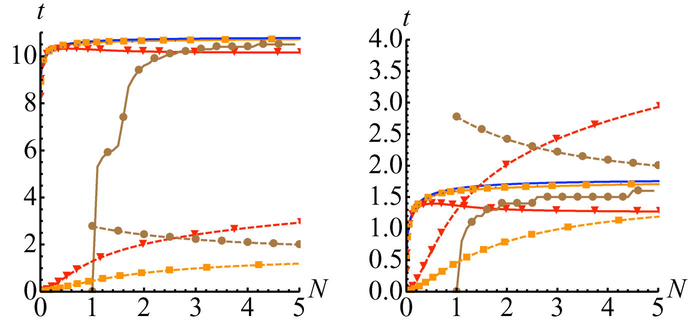

Fig. 2. (color on line) separation times t_S (straight lines) and initial nonGaussianities δ_0 (dashed lines) for $N_T = 10^{-5}$ (left) and $N_T = 10^{-1}$ (right) and for different PNES classes, PASV (brown, circle), PSSV (orange, square), TMC (red, triangle), TWB (blue) as a function of initial energy N.

nonGaussianities of different PNES are plotted against N (for t_S we use $t_M = \max_K t_K(\varrho)$) for $N_T = 10^{-5}$: at any fixed N, the states with higher δ_0 have shorter separation times. From the right panel of Fig. 2 we see that the same behaviour shows up also at low T (where t_{SI} is only a lower bound and the relation between t_S and δ_0 is not necessarily represented by (7)). This behaviour is connected with the fact that Gaussian states are maximally entangled at any fixed energy.[6,7] However, the relation between separation times, nonGaussianity and the initial entanglement of the states is not trivial, as we shall now discuss. As shown in Fig. 3, where t_S and δ_0 are plotted as a function of the initial entanglement ϵ_0, the dependence is by no means universal. However, we notice that also at fixed ϵ_0 states with higher δ_0 show shorter t_S: this trend is not represented by an exact relation, but it is a clear indication that nonGaussianity speeds up the loss of entanglement, making Gaussian entanglement more robust than nonGaussian one. The robustness of Gaussian entanglement can be conjectured to be a general feature of CV systems evolving in noisy Markovian channels. Indeed, within the Markovian approximation, propagation in CV noisy channels corresponds to a Master Equation in Linblad form, which induces a Gaussian map and enforces Gaussification of any initial state. The results of our analysis, together with the above discussions, naturally lead us to formulate the following general conjecture: *for any fixed value of the global energy of a PNES, and for any given noisy Markovian evolution with losses and thermal hopping, the Gaussian states are those that have maximal separation times.*

Besides, from Fig. 2 we also extract another very relevant feature: *in the high-energy limit and independently of the temperature there is an approximate universality in separation times* i.e. t_S are nearly constant and similar for all classes of states, nonGaussian ones being nearly as robust as Gaussian ones, i.e. the effect of the departure from Gaussianity is very small.

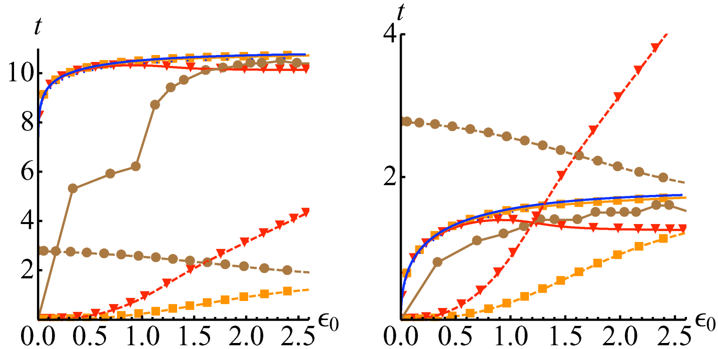

Fig. 3. (color on line) separation times t_S (straight lines) and initial nonGaussianities δ_0 (dashed lines) for $N_T = 10^{-5}$ (left) and $N_T = 10^{-1}$ (right) for different PNES classes, PASV (brown, circle), PSSV (orange, square), TMC (red, triangle), TWB (blue) as a function of ϵ_0.

6. Final Remarks

We have considered a class of states (PNES) including Gaussian and nonGaussian subclasses, assessing the robustness of their entanglement to different kinds of noise.

We have found that *all PNES are robust against phase diffusion noise*, i.e. entanglement is never destroyed for any finite time, and the same is true if mere losses are cosidered. *When both losses and thermal hopping are present, entanglement vanishes within a finite time* (*separation time*). To estimate the separation time, we have used several entanglement criteria: our analysis shows that no criterion is able to give better bounds than those provided by Simon's criterion, which is then optimal. At low temperature, estimates provided by Simon's criterion are very reliable since PNES gaussify well before they lose entanglement, whereas at high temperature they represent just lower bounds on separation times. *At any fixed energy, separation times decrease with the initial non-Gaussianity* δ_0, *both at high and at low temperature*. We conjecture that this feature holds in general for any Markovian evolution with losses and thermal hopping. Moreover, *for any fixed initial entanglement* ϵ_0 *separation time is longer for states with lower* δ_0, *i.e. Gaussian entanglement is the most robust against noise. Yet*, *in the high energy limit and independently of the temperature*, *the differences among separation times of different subclasses are small*, *nonGaussian states being nearly as robust as Gaussian ones.*

As a consequence, as long as PNES are considered, Gaussian states are optimal resources in terms of robustness to noise. On the other hand, our analysis shows that robustness of nonGaussian states is comparable with that of Gaussian states for sufficiently high energy of the states. This implies that in these regimes, nonGaussian resources can be exploited to improve quantum communication protocols approximately over the same distances.

References

1. M. Allegra *et al.*, quant-ph/1003.3331 (2010).
2. R. Horodecki *et al.*, *Rev. Mod. Phys.* **81**(2) (2009) 865.
3. B.-G. Englert and K. Wódkiewicz, *Int. J. Q. Inf.* **1** (2003) 153.
4. J. Eisert *et al.*, *Int. J. Quant. Inf.* **1** (2003) 479.
5. S. L. Braunstein *et al.*, *Rev. Mod. Phys.* **77** (2005) 513.
6. A. Ferraro *et al.*, *Gaussian States in Quantum Information* (Bibliopolis, Napoli, 2005).
7. F. Dell'Anno *et al.*, *Phys. Rep.* **428** (2006) 53.
8. G. Giedke and J. I. Cirac, *Phys. Rev. A* **66** (2002) 032316.
9. J. Eisert *et al.*, *Phys. Rev. Lett.* **89** (2002) 137903.
10. J. Fiurasek, *Phys. Rev. Lett.* **89** (2002) 137904.
11. R. Dong *et al.*, *Nature Phys.* **4** (2008) 919.
12. H. Takahashi *et al.*, *Nature Phot.* **4** (2010) 178.
13. F. Casagrande *et al.*, *Phys. Rev. A* **75** (2007) 032336.
14. N. J. Cerf *et al.*, *Phys. Rev. Lett.* **95** (2005) 070501.
15. T. Opatrny *et al.*, *Phys. Rev. A* **61** (2000) 032302.
16. P. T. Cochrane *et al.*, *Phys. Rev. A* **65** (2002) 062306.
17. S. Olivares *et al.*, *Phys. Rev. A* **67** (2003) 032314.

18. F. Dell'Anno *et al.*, *Phys. Rev. A* **81** (2010) 012333.
19. O. Ayhür and P. Kumar, *Phys. Rev. Lett.* **65** (1990) 1551.
20. O. Haderka *et al.*, *Phys. Rev. A* **71** (2005) 033815.
21. K. Hayasaka *et al.*, *Opt. Lett.* **29** (2004) 1665.
22. J. Laurat *et al.*, *Phys. Rev. Lett.* **91** (2003) 213601.
23. A. C. Funk and M. G. Raymer, *Phys. Rev. A* **65** (2001) 042307.
24. L. Mista Jr. *et al.*, *Phys. Rev. A* **65** (2001) 062315.
25. V. C. Usenko and B. I. Lev, *Phys. Lett. A* **348** (2005) 17.
26. V. C. Usenko and M. G. A. Paris, *Phys. Rev. A* **75** (2007) 043812.
27. Y. Zhang *et al.*, *Opt. Exp.* **11** (2003) 3592.
28. M. Lewenstein *et al.*, *Phys. Rev. A* **62** (2000) 052310.
29. M. Lewenstein, *Phys. Rev. A* **63** (2001) 044304.
30. D. Bruss *et al.*, *J. Mod. Opt.* **49** (2002) 1399.
31. M. Dakna *et al.*, *Phys. Rev. A* **55** (1997) 3184.
32. Z. X. Zhang and H. Y. Fan, *Phys. Lett. A* **165** (1992) 14.
33. J. Wenger *et al.*, *Phys. Rev. Lett.* **92** (2004) 153601.
34. M. S. Kim *et al.*, *Phys. Rev. Lett.* **101** (2008) 260401.
35. A. Zavatta *et al.*, *Phys. Rev. A* **75** (2007) 052106.
36. G. S. Agarwal, *Phys. Rev. Lett.* **57** (1986) 827.
37. G. S. Agarwal, *J. Opt. Soc. Am. B* **5** (1988) 1940.
38. G. Lindblad, *Commun. Math. Phys.* **48** (1976) 119.
39. D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2006).
40. G. M. D'Ariano, *Phys. Lett. A* **187** (1994) 231.
41. G. M. D'Ariano *et al.*, *Phys. Rev. A* **67** (2003) 042310.
42. R. Simon, *Phys. Rev. Lett.* **84** (2000) 2726.
43. A. Serafini *et al.*, *J. Opt. B* **7** (2005) R19-R36.
44. E. Shchukin and W. Vogel, *Phys. Rev. Lett.* **95** (2005) 230502.
45. A. Miranowicz and M. Piani, *Phys. Rev. Lett.* **97** (2006) 058901.
46. M. Horodecki *et al.*, *Phys. Lett. A* **223** (1996) 1.
47. J. Sperling and W. Vogel, *Phys. Rev. A* **79** (2009) 022318.
48. K. Chen and L. Wu, *Quant. Inf. and Comp.* **3** (2003) 193.
49. O. Rudolph, *J. Phys. A* **33** (2000) 3951.
50. M. G. Genoni *et al.*, *Phys. Rev. A* **78** (2008) 060303.
51. V. Vedral, *Rev. Mod. Phys.* **74** (2002) 197.
52. M. M. Wolf *et al.*, *Phys. Rev. Lett.* **96** (2006) 080502.