

**Coevolutionary network approach to cultural dynamics controlled by intolerance**Carlos Gracia-Lázaro,<sup>1,2</sup> Fernando Quijandría,<sup>3</sup> Laura Hernández,<sup>3</sup> Luis Mario Floría,<sup>1,2</sup> and Yamir Moreno<sup>1,4,5</sup><sup>1</sup>*Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza E-50018, Spain*<sup>2</sup>*Departamento de Física de la Materia Condensada, University of Zaragoza, Zaragoza E-50009, Spain*<sup>3</sup>*Laboratoire de Physique Théorique et Modélisation, UMR CNRS, Université de Cergy-Pontoise, 2 Avenue Adolphe Chauvin, F-95302, Cergy-Pontoise Cedex, France*<sup>4</sup>*Departamento de Física Teórica, University of Zaragoza, Zaragoza E-50009, Spain*<sup>5</sup>*Complex Networks and Systems Lagrange Lab, Institute for Scientific Interchange, Viale S. Severo 65, I-10133 Torino, Italy*

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Starting from Axelrod's model of cultural dissemination, we introduce a rewiring probability, enabling agents to cut the links with their unfriendly neighbors if their cultural similarity is below a tolerance parameter. For low values of tolerance, rewiring promotes the convergence to a frozen monocultural state. However, intermediate tolerance values prevent rewiring once the network is fragmented, resulting in a multicultural society even for values of initial cultural diversity in which the original Axelrod model reaches globalization.

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**I. INTRODUCTION**

The growing interest in the interdisciplinary physics of complex systems has focused physicists' attention on agent-based modeling [1,2] of social dynamics as a very attractive methodological framework for social sciences, where concepts and tools from statistical physics turn out to be very appropriate [3] for the analysis of the collective behaviors emerging from the social interactions of the agents. The dynamical social phenomena of interest include residential segregation [4,5], cultural globalization [6,7], opinion formation [8,9], rumor spreading [10,11], and others.

The question that motivates the formulation of Axelrod's model for cultural dissemination [6] is how cultural diversity among groups and individuals could survive despite the tendencies to become more and more alike as a result of social interactions. The model assumes a highly nonbiased scenario, where the culture of an agent is defined as a set of equally important cultural features, whose particular values (traits) can be transmitted (by imitation) among interacting agents. It also assumes that the driving force of cultural dynamics is the "homophile satisfaction," the agents' commitment to become more similar to their neighbors. Moreover, the more cultural features an agent shares with a neighbor, the more likely the agent will imitate an uncommon feature's trait of the neighbor agent. In other words, the higher the cultural similarity, the higher the social influence.

The simulations of the model dynamics show that for low initial cultural diversity, measured by the number  $q$  of different traits for each cultural feature (see below), the system converges to a global cultural state, while for  $q$  above a critical value  $q_c$  the system freezes in an absorbing state where different cultures persist. The (nonequilibrium) phase transition [12] between globalization and multiculturalism was first studied for a square planar geometry [7,13,14], but soon other network structures of social links [15–17] were considered, as well as the effects of different types of noise ("cultural drift") [18,19], external fields (modeling, e.g., influential media, or information feedback) [20–23], and global or local nonuniform couplings [24,25].

In all those extensions of Axelrod's model mentioned in the above paragraph, the cultural dynamics occurs on a network of social contacts that is fixed from the outset. However, very often social networks are dynamical structures that continuously reshape. A simple mechanism of network reshaping is agents' mobility, and a scenario (named the Axelrod-Schelling model) where cultural agents placed in culturally dissimilar environments are allowed to move has recently been analyzed [26,27]. In this model, new interesting features of cultural evolution appear depending on the values of a parameter, the (in)tolerance, that controls the strength of agents' mobility.

A different mechanism of network reshaping has been considered in Refs. [28,29], where a cultural agent breaks its link to a completely dissimilar neighbor, redirecting it to a randomly chosen agent. At variance with the mobility scenario of the Axelrod-Schelling model, that limits the scope of network structures to clusters' configurations on the starting structure (square planar lattice, or others), the rewiring mechanism allows for a wider set of network structures to emerge in the coevolution of culture and social ties [30].

In this Brief Report, we introduce in the scenario of network rewiring a tolerance parameter  $Z$  controlling the likelihood of links rewiring, in such a way that the limit  $Z = 1^-$  recovers the case analyzed in Refs. [28,29], where only links with an associated null cultural overlap are broken. Lower values of  $Z$  correspond to less tolerant attitudes, where social links with progressively higher values of the cultural overlap may be broken with some probability that depends on these values. The results show a counterintuitive dependence of the tolerance  $Z$  on the critical value  $q_c$ . On one hand, as expected from [28,29], rewiring promotes globalization for high values of the tolerance, but, on the other hand, very low values of  $Z$  (which enhance the rewiring probability) show the higher values of  $q_c$ . Indeed, a nonmonotonous behavior is observed in  $q_c(Z)$ : our results unambiguously show that for some intermediate values of the tolerance  $Z$ , cultural globalization is disfavored with respect to the original Axelrod's model where no rewiring of links is allowed. In other words, rewiring does not always promote globalization.

On the other hand, the resulting network topology depends on  $q$ , changing from a Poisson connectivity distribution  $P(k)$  to a fat tailed distribution for  $q \sim q_c$ .

## II. MODEL

As in Axelrod's model, the culture of an agent  $i$  is a vector of  $F$  integer variables  $\{\sigma_f(i)\}$  ( $f = 1, \dots, F$ ), called *cultural features*, that can take on  $q$  values,  $\sigma_f(i) = 0, 1, \dots, q - 1$ , the *cultural traits* that the feature  $f$  can assume. The  $N$  cultural agents occupy the nodes of a network of average degree  $\langle k \rangle$  whose links define the social contacts among them. The dynamics is defined, at each time step, as follows.

(i) Each agent  $i$  imitates an uncommon feature's trait of a randomly chosen neighbor  $j$  with a probability equal to their *cultural overlap*  $\omega_{ij}$ , defined as the proportion of common cultural features,

$$\omega_{ij} = \frac{1}{F} \sum_{f=1}^F \delta_{\sigma_f(i), \sigma_f(j)}, \quad (1)$$

where  $\delta_{x,y}$  denotes the Kronecker's delta, which is 1 if  $x = y$  and 0 otherwise. The whole set of  $N$  agents perform this step in parallel.

(ii) Each agent  $i$  disconnects its link with a randomly chosen neighbor agent  $j$  with probability equal to its *dissimilarity*  $1 - \omega_{ij}$ , provided the dissimilarity  $1 - \omega_{ij}$  exceeds a threshold (*tolerance*)  $Z$ ,

$$1 - \omega_{ij} > Z, \quad (2)$$

and rewires it randomly to other non-neighbor agent. The tolerance  $0 \leq Z \leq 1$  is a model parameter.

First, we note that the initial total number of links in the network is preserved in the rewiring process, so the average degree  $\langle k \rangle$  remains constant. However, the rewiring process allows for substantial modifications of the network topological features, e.g., connectedness, degree distribution, etc. In that respect, except for the limiting situation of very low initial cultural diversity  $q$  and a very high tolerance  $Z$  (where the likelihood of rewiring could be very low), one should expect that the choices for the initial network of social ties have no influence in the asymptotic behavior of the dynamics.

When the threshold tolerance  $Z$  satisfies  $\frac{F-1}{F} \leq Z < 1$ , only those links among agents with zero cultural overlap are rewired, so the model becomes the one studied in Refs. [28,29]. On the other hand, when the tolerance takes the value  $Z = 1$ , there is no rewiring likelihood and the original Axelrod's model is recovered. When  $Z = 0$ , rewiring is always possible provided the cultural similarity is not complete, i.e.,  $\omega_{ij} \neq 1$ , so that it corresponds to the highest intolerance.

The usual order parameter for Axelrod's model is  $\langle S_{\max} \rangle / N$ , where  $\langle S_{\max} \rangle$  is the average (over a large number of different random initial conditions) of the number of agents sharing the most abundant (dominant) culture, and  $N$  is the number of agents in the population. Large values of the order parameter characterize the globalization (cultural consensus) regime. We also compute the normalized size  $\langle S_{\text{top}} \rangle / N$  of the largest network component (i.e., the largest connected subgraph of the network).

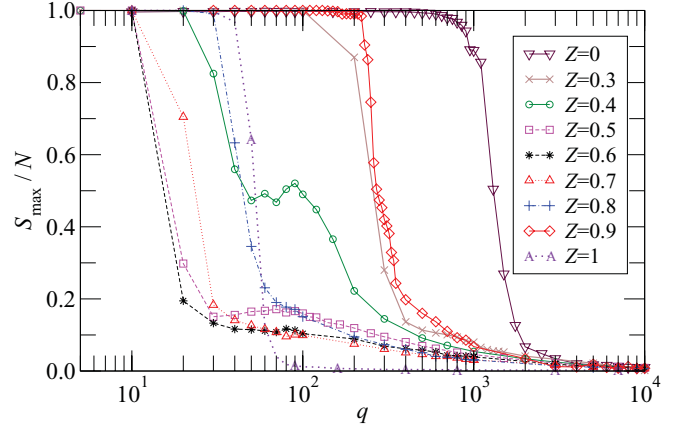


FIG. 1. (Color online) Order parameter as a function of the variability  $q$  for different values of the tolerance threshold  $Z$ .  $N = 900$  and  $\langle k \rangle = 4$ , averaged over 1000 replicas.

## III. RESULTS AND DISCUSSION

We have studied networks of sizes  $N = 900$  and  $1600$ , averaging over 50–2000 replicas. Checks for robustness of main results with larger size of  $N = 2500$  were also made. The considered cultural vectors have  $F = 10$  cultural features, each one with a variability of  $q = 5$ –10000. We studied different values of the tolerance threshold  $Z \in (0, 1)$  and different values of the average connectivity  $\langle k \rangle = 4, 10, 20, 40$ . Each simulation is performed for  $N$ ,  $F$ ,  $\langle k \rangle$ ,  $Z$ , and  $q$  fixed. For the sake of comparison with previous results [28,29], we will present results for  $\langle k \rangle = 4$ .

The behavior of the order parameter for different values of  $Z$  is seen in Fig. 1. As in Ref. [28], three different macroscopic phases are observed with increasing values of  $q$ , namely a monocultural phase, with a giant cultural cluster, a multicultural one with disconnected monocultural domains, and finally a multicultural phase with continuous rewiring. The nature of the latter phase has been successfully explained in Ref. [28]: at very large values of the initial cultural diversity  $q$ , the expected number of pairs of agents sharing at least one cultural trait becomes smaller than the total number of links in the network, so that rewiring cannot stop. Here we will focus attention on the first two phases and the transition between them.

In Fig. 2, we show the size distribution of the dominant culture over different realizations, measured for different values of  $q$ , at a particular fixed value of the tolerance  $Z = 0.5$ . In the region of  $q$  values near the transition from globalization to multiculturalism, the distribution is double peaked, indicating that the transition is first order, as in the original Axelrod's model. The transition value,  $q_c$ , may be roughly estimated as the  $q$  value, where the areas below the peaks of the size distribution are equal. The estimates of the transition points for different values of the tolerance  $Z$  are shown in Fig. 3. The nonmonotonous character of the graph  $q_c(Z)$  seen in this figure reveals a highly nontrivial influence of the tolerance parameter on the coevolution of cultural dynamics and the network of social ties.

Let us first consider the (most tolerant) case  $Z = 0.9$  that, except for the system size  $N$ , coincides exactly with the

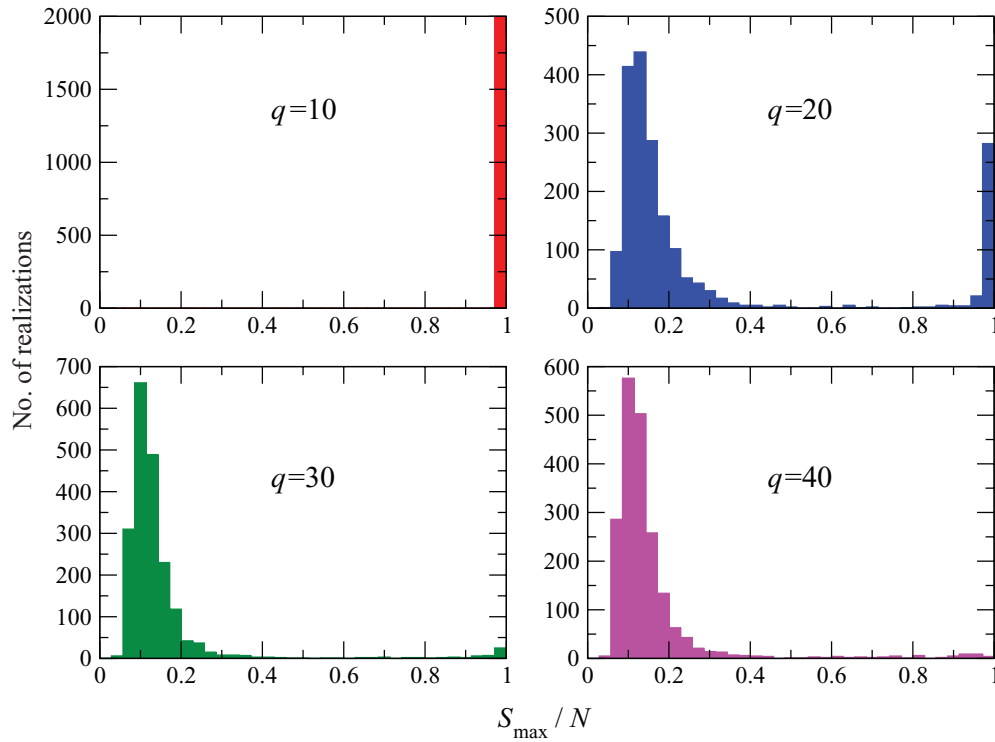


FIG. 2. (Color online) Histograms of  $S_{\max}/N$  for different values of  $q$ , and for a fixed tolerance  $Z = 0.5$ ,  $N = 900$ , and  $\langle k \rangle = 4$ . From this figure, one gets  $q_c \sim 20$ .

situation considered in Ref. [29], i.e., only links with zero cultural overlap are rewired. As discussed in Ref. [29], for  $q$  values larger than the critical value for a fixed network [ $q_c(Z = 1) \simeq 60$ ], rewiring allows redirecting links with zero overlap to agents with some common cultural trait (compatible agents), so reinforcing the power of social influence to reach cultural globalization. Once all links connect compatible agents, rewiring stops [31]. From there on, the network structure will remain fixed, and globalization will be reached with the proviso that the network has so far remained connected. This is the case for most realizations (for  $N = 900$ ) up to values of

$q \sim 240$ . Increasing further the cultural diversity  $q$  increases the frequency of rewiring events and slows down the finding of compatible agents, favoring the topological fragmentation into network components before rewiring stops. Under these conditions, the asymptotic state will consist of disconnected monocultural components.

On one hand, network plasticity allows one to connect compatible agents, so promoting globalization; but, on the other hand, it may produce network fragmentation, so favoring multiculturalism. What we have seen in the previous paragraph is that, for  $Z = 0.9$ , the first effect prevails over the second one up to  $q_c(Z = 0.9) \simeq 240$ . Going from there to less tolerant situations (decreasing  $Z$ ) increases the likelihood of rewiring, making it easier that network fragmentation occurs before rewiring stops. This has the effect of decreasing the critical value  $q_c$ . In fact, from Fig. 3, we see that for  $Z = 0.7, 0.6$ , and  $0.5$  multiculturalism prevails for cultural diversities where the original Axelrod’s model shows cultural globalization. In these cases, network plasticity promotes multiculturalism in a very efficient way: Agents segregate from neighbors with low cultural similarity and form disconnected social groups where full local cultural consensus is easily achieved, for  $q$  values low enough to allow a global culture in fixed connected networks.

For very low values of the tolerance parameter, though network fragmentation occurs easily during the evolution, Fig. 3 shows that globalization persists up to very high values of the initial cultural diversity  $q$ . To explain this seemingly paradoxical observation, one must realize that network fragmentation is not an irreversible process, provided links connecting agents with high cultural overlap have a positive

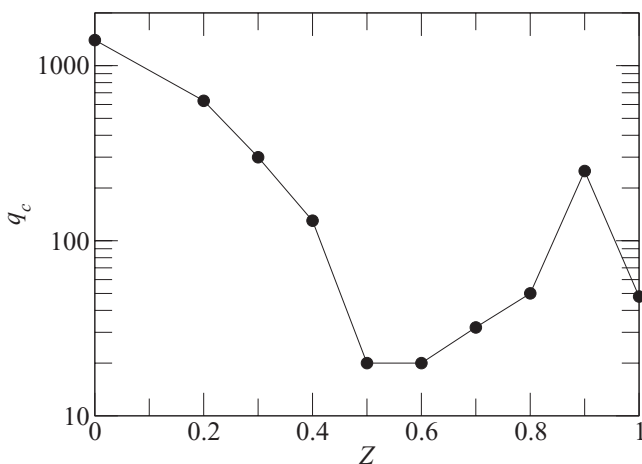


FIG. 3. Critical value of the diversity  $q_c$  vs the tolerance threshold  $Z$ , obtained from the distribution of sizes of the dominant culture.  $N = 900$  and  $\langle k \rangle = 4$ . See the text for further details.

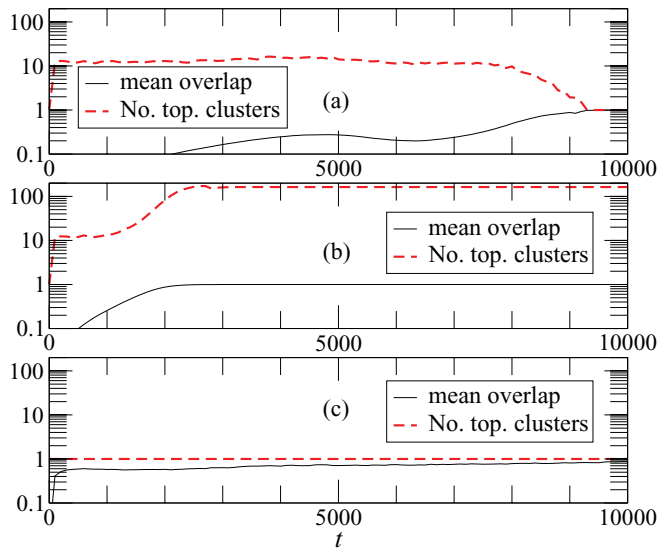


FIG. 4. (Color online) Time evolution of mean overlap and number of topological clusters for different values of tolerance  $Z = 0.2$  (a),  $Z = 0.6$  (b), and  $Z = 0.9$  (c).  $N = 900$  and  $q = 100$ . See the text for further details.

rewiring probability. Under these circumstances, transient connections among different components occur so frequently so as to make possible a progressive cultural homogenization between components that otherwise would have separately reached different local consensuses. Figure 4 illustrates the time evolution for  $q = 100$  and different values of  $Z$ . Panel (a) shows an example of cultural evolution where network fragmentation reverts to a connected monocultural network for  $Z = 0.2$ . Panel (b), that corresponds to  $Z = 0.6$ , shows that social fragmentation persists during the whole evolution, while in panel (c), which corresponds to the most tolerant situation ( $Z = 0.9$ ), the network remains connected all the time.

The degree distribution of the network is Poissonian centered about  $\langle k \rangle$  for all  $q$  values, except for  $q \gtrsim q_c$ , where it becomes fat tailed, with several lowly connected (and disconnected) sites. For very high  $q$  values, in the dynamical

phase, the network rewiring is essentially random, so  $P_q(k)$  is again Poisson like, centered around  $\langle k \rangle$ .

#### IV. SUMMARY

In this Brief Report, we have generalized the scenario for coevolution of Axelrod's cultural dynamics and network of social ties that was considered in Refs. [28,29], by introducing a tolerance parameter  $Z$  that controls the strength of network plasticity. Specifically,  $Z$  fixes the fraction of uncommon cultural features above which an agent breaks its tie with a neighbor (with probability equal to the cultural dissimilarity), so that the lower the  $Z$  value, the higher the social network plasticity.

Our results show that the network plasticity, when controlled by the tolerance parameter, has competing effects on the formation of a global culture. When tolerance is highest, network plasticity promotes cultural globalization for values of the initial cultural diversity where multiculturalism would have been the outcome for fixed networks. On the contrary, for intermediate values of the tolerance, the network plasticity produces the fragmentation of the (artificial) society into disconnected cultural groups for values of the initial cultural diversity where global cultural consensus would have occurred in fixed networks. For very low values of the tolerance, social fragmentation occurs during the system evolution, but the network plasticity is so high that it allows the final cultural homogenization of the transient groups for very high values of the cultural diversity. Intermediate tolerances promote multiculturalism, while both extreme intolerance and extreme tolerance favor the formation of a global culture, with the former being more efficient than the latter.

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