

Suppression of Friction by Mechanical Vibrations

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Mechanical vibrations are known to affect frictional sliding and the associated stick-slip patterns causing sometimes a drastic reduction of the friction force. This issue is relevant for applications in nanotribology and to understand earthquake triggering by small dynamic perturbations. We study the dynamics of repulsive particles confined between a horizontally driven top plate and a vertically oscillating bottom plate. Our numerical results show a suppression of the high dissipative stick-slip regime in a well-defined range of frequencies that depends on the vibrating amplitude, the normal applied load, the system inertia and the damping constant. We propose a theoretical explanation of the numerical results and derive a phase diagram indicating the region of parameter space where friction is suppressed. Our results allow to define better strategies for the mechanical control of friction.

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Natural or artificially induced manipulations by small mechanical vibrations, when applied at suitable frequency and amplitude ranges, may help in driving a contacting sliding interface out of its potential energy minima, thus increasing considerably surface mobility and diffusion, and reducing friction. This has been shown experimentally for sliding of nanoscale contacts through, e.g., the atomic force microscope [1–3], and in computer simulations via extended molecular dynamics [4] and simple modeling approaches [5–7]. On a larger scale, it has been observed that in sheared granular media experiments the stick-slip behavior is significantly perturbed by tiny transverse vibrations [8,9]. Since geological faults are often filled with a granular gouge, these results might be relevant to understand earthquake triggering by low amplitude seismic waves [10]. Despite these promising numerical and experimental contributions, a quantitative theory accounting for the friction dependence on vibrations is still lacking.

In this Letter, we study the frictional properties of a two dimensional set of repulsive particles confined between two rigid plates (see Fig. 1) [11,12]. The top plate is attached to a spring that is pulled at constant velocity, while the bottom plate is vibrated vertically. Without vibration, the top plate exhibits a characteristic stick-slip behavior. Vibrations induce a drastic reduction of the friction coefficient and a suppression of the stick-slip, but only in a well-defined frequency range for the mechanical oscillation. The robustness of this result is checked by changing a large set of parameters, such as the damping constant, the load and the number of confined layers. Next, we propose a theoretical argument that allows to determine the general conditions under which friction is suppressed. The theory is able to reproduce quantitatively all our numerical results *without any fitting parameters*. The gen-

erality of the theory and its quantitative agreement with the numerical simulations suggests that it could easily be adapted to different cases, providing a way to predict the effect of vibration on friction, with far-ranging implications from nanotribology to geophysics.

Our two dimensional system consists of two identical top and bottom substrates, composed of $n_t = n_b$ particles with coordinates \mathbf{r}_i^t and \mathbf{r}_i^b , respectively, and constant lattice separation $a_s = 1$. We confine n_p particles with coordinates \mathbf{r}_i^p between the top and bottom plates. The mass m of all the particles is the same so that the total mass of the confined layers is $M_p = mn_p$ while the top and bottom plate have a mass $M_{\text{bot}} = M_{\text{top}} = mn_t$. Because of the vertical confinement, periodic boundary conditions are applied only along the x direction. The particles interact via a pairwise repulsive potential $U(r) = U_0[(\frac{r_0}{r})^{12} - 2(\frac{r_0}{r})^6]$ for $r < r_0$ and $U(r) = 0$ otherwise. The parameters of the potential are the same for all the particles, whether in the top and bottom plate or in between them. We adjust the number of particles n_p , imposing that $n_p = N_t \cdot n_t = N_b \cdot n_b$ with N_t an integer number. In this way, the system forms

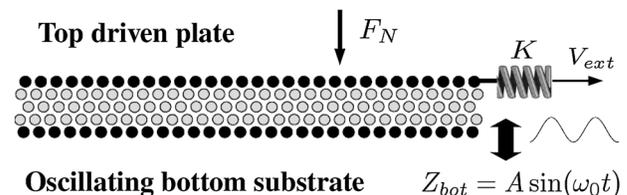


FIG. 1. Sketch of the system, with the rigid top and bottom plates indicated in black and the confined particles in gray. The top plate is dragged through a spring of elastic constant K moving at constant velocity V_{ext} , while the bottom plate vibrates vertically with frequency ω_0 and amplitude A .

N_l perfectly ordered layers of confined particles. We also consider the case of a few missing particles, finding exactly the same results as long as the system remains ordered. The top plate is subject to a normal force F_N and is pulled through a spring, of elastic constant K , which moves along the horizontal x direction with constant velocity V_{ext} .

Indicating with $\mathbf{R}_{\text{top}} = (X_{\text{top}}, Z_{\text{top}})$ and $\mathbf{R}_{\text{bot}} = (X_{\text{bot}}, Z_{\text{bot}})$ the center of mass coordinates of the top and bottom plate, respectively, where $\mathbf{R}_{\text{top}} = \frac{1}{n_t} \sum_{i=1}^{n_t} \mathbf{r}_i^t$, and $\mathbf{R}_{\text{bot}} = \frac{1}{n_b} \sum_{i=1}^{n_b} \mathbf{r}_i^b$, the particles and the top plate satisfy the equations of motion

$$m\ddot{\mathbf{r}}_i^p + \sum_{i \neq j} \frac{d}{d\mathbf{r}_i} U(|\mathbf{r}_i^p - \mathbf{r}_j^p|) + m\eta(\dot{\mathbf{r}}_i^p - \dot{\mathbf{R}}_{\text{top}}) + m\eta(\dot{\mathbf{r}}_i^p - \dot{\mathbf{R}}_{\text{bot}}) + \mathbf{f}^{\text{ran}} = 0 \quad (1)$$

$$M_{\text{top}}\ddot{X}_{\text{top}} + \sum_{i=1}^{n_t} \sum_{j=1}^{n_p} \frac{d}{dx_i^t} U(|\mathbf{r}_j^p - \mathbf{r}_i^t|) + K(X_{\text{top}} - V_{\text{ext}}t) + \sum_{i=1}^{n_p} m\eta(\dot{X}_{\text{top}} - \dot{x}_i^p) + f_x^{\text{ran}} = 0 \quad (2)$$

$$M_{\text{top}}\ddot{Z}_{\text{top}} + \sum_{i=1}^{n_t} \sum_{j=1}^{n_p} \frac{d}{dz_i^t} U(|\mathbf{r}_j^p - \mathbf{r}_i^t|) + F_N + \sum_{i=1}^{n_p} m\eta(\dot{Z}_{\text{top}} - \dot{z}_i^p) + f_z^{\text{ran}} = 0 \quad (3)$$

where $N = n_p + n_t + n_b$ and η is the damping coefficient that accounts for a viscous dissipation. The temperature is controlled by a Langevin thermostat according to the relation $\langle \mathbf{f}(t)^{\text{ran}} \mathbf{f}(t')^{\text{ran}} \rangle = 4m\eta k_B T \delta(t - t')$. In the present simulations, we consider very low temperatures $k_B T = 10^{-2} U_0$.

To study the influence of mechanical vibrations on the system, the bottom plate is vibrated vertically $Z_{\text{bot}} = Z_0 + A \sin(\omega_0 t)$ where Z_0 is a reference coordinate, A is the amplitude and ω_0 the frequency, while its horizontal component X_{bot} is held fixed. In all the simulations, we compute the instantaneous friction force F_L by measuring the spring elongation of the driving apparatus, $(X_{\text{top}}(t) - V_{\text{ext}}t)$ so that $F_L = K[X_{\text{top}}(t) - V_{\text{ext}}t]$. The friction coefficient is defined as $\mu \equiv F_L/F_N$ and its average $\langle \mu \rangle$, obtained integrating its value over a sufficiently long time interval in the steady state.

In Fig. 2(a), we show the behavior of the friction coefficient μ as a function of time in the case of three confined particle layers for three distinct values of the frequency ω_0 . For $\omega_0 = 1$ and $\omega_0 = 3$, we observe a characteristic stick-slip behavior, with loading phases where the system is stuck, followed by rapid slip events in which the force accumulated by the spring is relaxed. A very similar pattern also takes place in absence of vibrations at low external driving rates, with the system alternately sticking and

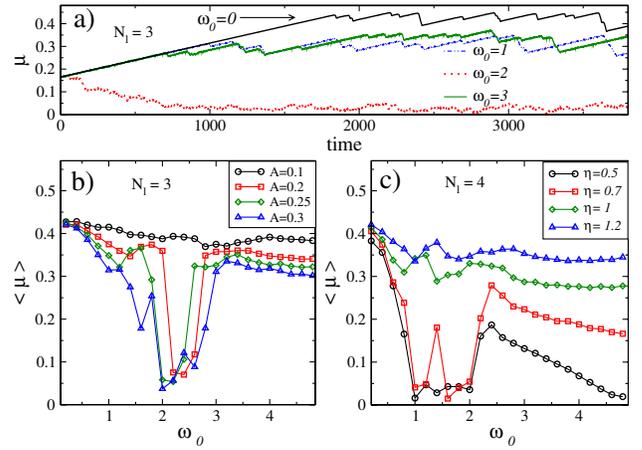


FIG. 2 (color online). (a) Friction coefficient μ vs time obtained for three values of the bottom oscillating frequency ω_0 (three lubricant layers). (b) Time-averaged value of μ as a function of ω_0 for four values of the oscillation amplitude A (ranging from 3% to 9% of the $N_l = 3$ lubricant thickness). (c) Time-averaged value of μ vs ω_0 for $N_l = 4$ and four values of the damping coefficient η .

slipping forward (not shown). At $\omega_0 = 2$, we observe a strong reduction of the friction coefficient and a drastic suppression of the sawtooth stick-slip behavior. To make this observation more quantitative, we report in Fig. 2(b) and 2(c) the systematic variations of the average friction coefficient $\langle \mu \rangle$ with the vibration frequency ω_0 . The left panel shows results obtained for various oscillation amplitudes A , ranging from 3% to 9% of the film thickness, while the right panel displays results for different values of the damping coefficient η . We see that the suppression of friction appears in a well-defined range of frequencies $[\omega_1, \omega_2]$, which depends on A and η .

To understand the origin of this phenomenon, we compute the power spectrum of the vertical position of the top substrate Z_{top} for the three values of ω_0 corresponding to Fig. 2(a). At low frequencies [inset in Fig. 3(a)], the top plate and the confined particles vibrate in phase with the oscillations of the bottom plate. Hence, the spectrum displays a peak at frequency ω_0 and its integer multiples and vibrations have no visible effect on sliding friction. The situation is drastically different at $\omega_0 = 2$ [Fig. 3(b)]. In this case, the top plate and the confined particles can not follow the bottom plate. The vertical position of the top plate increases, presenting high amplitude oscillations which diminish the contact time between the confined particles and the bottom plate, reducing considerably the friction force. The corresponding Fourier spectrum shows additional peaks at integer multiples of $\omega_0/2$. We find that this is a general feature of the spectrum in the interval of friction suppression $[\omega_1, \omega_2]$. Additional peaks may appear at frequencies $\omega = \omega_0/n_0$, where n_0 is an integer, implying that the top plate oscillates with a period that is an integer multiple of the driving period on the bottom plate. Further increases of the oscillation frequency induce a

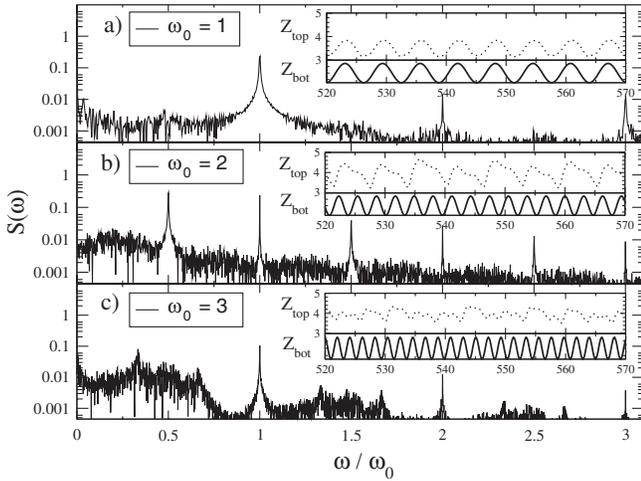


FIG. 3. Power spectrum of the vertical component Z_{top} of the plate for different values of the oscillating frequency ω_0 and $N_l = 3$, $A = 0.3$, $\eta = 1$. At low-frequency ($\omega_0 = 1$), the system follows the oscillating bottom substrate. At higher frequencies $\omega_0 = 2$, the system oscillates also at one-half frequency of ω_0 . At $\omega_0 = 3$, a low-frequency noise appears. In the insets, we report the time evolution of Z_{top} (dotted line) and Z_{bot} (thick line).

reduction of the amplitude of the top plate oscillations around the equilibrium position [see Fig. 3(c)] so that the low-frequency noise suppresses the resonant peaks. As a result, the friction force increases again.

The mechanism illustrated by the spectral analysis suggests an argument to derive analytically the values of ω_1 and ω_2 , defining the interval of friction suppression. At low frequencies, the particles follow coherently the oscillation of the bottom plate owing to the downward action of the normal load F_N . The reduction of friction occurs when the particles are able to detach from the bottom plate due to the action of the inertial force induced by the external vibration $F_{\text{in}} = M\ddot{Z}_{\text{bot}}$, where $M = M_p + M_{\text{top}}$. The frequency ω_1 corresponds to the condition for which the inertial force overcomes the combined action of the normal load F_N and the damping force $F_{\text{damp}} = M_p\eta\dot{Z}_{\text{bot}}$

$$F_{\text{in}}(\omega_1) \simeq F_N(\omega_1) + F_{\text{damp}}(\omega_1). \quad (4)$$

Estimating the inertial force as $F_{\text{in}} \simeq MA\omega_0^2$ and the damping force as $F_{\text{damp}} \simeq M_p\eta A\omega_0$, we obtain an implicit expression for the starting frequency of friction suppression ω_1

$$MA\omega_1^2 = F_N + M_p\eta A\omega_1. \quad (5)$$

It is convenient to work with dimensionless quantities, defining

$$\tilde{f} \equiv \frac{F_N}{MA\eta^2} \quad \tilde{m} \equiv \frac{M_p}{M} \quad \tilde{\omega} \equiv \frac{\omega}{\eta}. \quad (6)$$

Using these rescaled variables, Eq. (5) yields

$$\tilde{\omega}_1 = \frac{1}{2}(\tilde{m} + \sqrt{\tilde{m}^2 + 4\tilde{f}}). \quad (7)$$

To estimate the recovery frequency ω_2 , we determine the conditions for the presence of low-frequency vibrations. Because of the external oscillations, the confined layers detach from the bottom substrate during a characteristic time Δt that can be estimated as

$$\Delta t \simeq \dot{Z}_{\text{bot}}M/F_N \simeq A\omega_0M/F_N. \quad (8)$$

To observe low-frequency vibrations, which will cause the recovery of the friction force, the period of the external oscillation should be smaller than the rise time associated with the internal vibrations of the particles (i.e., $\frac{2\pi}{\omega_0} < \Delta t$). This condition corresponds to the maximum of the momentum transfer from the vibrating plate to the confined particles. Using again the dimensionless variables defined in Eq. (6), we estimate

$$\tilde{\omega}_2 = \sqrt{2\pi\tilde{f}}. \quad (9)$$

The theoretical predictions for ω_1 and ω_2 are in excellent agreement with the numerical simulations, as shown in Figs. 4(a) and 4(b). The numerical values are obtained varying the number of layers N_l , the vibration amplitude A , the damping coefficient η , and the normal load F_N . Notice that the theory has no adjustable parameters.

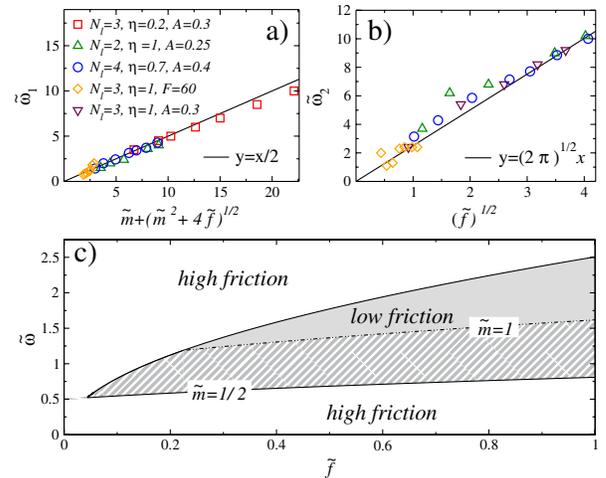


FIG. 4 (color online). (a) Comparison between the numerical results (symbols) and the theory (solid line). $\tilde{\omega}_1$ as a function of the dimensionless variables \tilde{f} and \tilde{m} . (b) $\tilde{\omega}_2$ as a function of \tilde{f} . The behaviors corresponding to different symbols are obtained keeping three parameters fixed and varying the fourth. (c) Phase diagram indicating the region of friction suppression. The region depends on the value of \tilde{m} that by construction lies in the interval $[\frac{1}{2}, 1]$. To illustrate this, we indicate in gray the region where friction is always suppressed, corresponding to the region above the line $\omega_0 = \omega_1(\tilde{f}, \tilde{m} = 1)$ and below the line $\omega_0 = \omega_2$. In the gray-striped region friction is suppressed only in some cases, depending on the value of \tilde{m} . Finally, in the white region, where $\omega_0 > \omega_2$ or $\omega_0 < \omega_1(\tilde{f}, \tilde{m} = 1/2)$, friction is never suppressed.

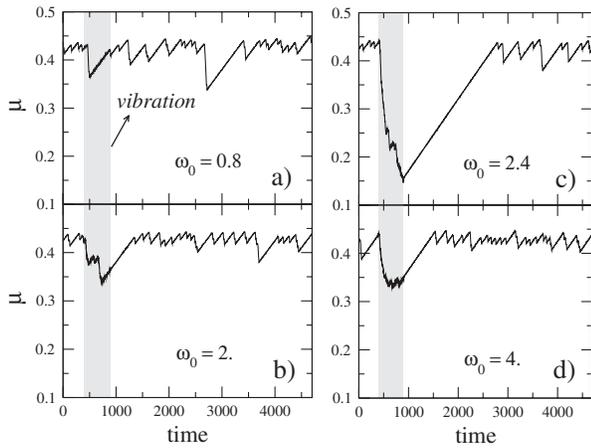


FIG. 5. The effect of a short time vibration pulse on the stick-slip behavior for different frequencies ω_0 , $N_l = 3$, $A = 0.2$. The shaded light gray regions indicate the duration of vibration pulse. The largest effect is visible when the frequency lies in the range of $\omega_1 < \omega_0 < \omega_2$.

From the analytical relations (7) and (9), we can draw a phase diagram indicating, in the space of dimensionless variables (6), the region where friction is suppressed [Fig. 4(c)]. The region of friction reduction is enclosed between $\tilde{\omega}_1$ and $\tilde{\omega}_2$ and shrinks as we reduce \tilde{f} , until it finally disappears. Notice that $\tilde{\omega}_1$ depends on the value of reduced mass which lies between $\tilde{m} = 1/2$, for a single confined layer, and $\tilde{m} = 1$ for an infinitely wide system. The possible values for the ω_1 are indicated by the gray-striped region in Fig. 4(c).

It is important to check the robustness of these results against modification of the model. To this end, we replace the interparticle potential with an attractive one. If the interaction between particles and substrate remains repulsive, we find no quantitative changes to the results. On the other hand, the frequency range for friction suppression is shifted to higher values when the particles are attracted by the substrate. This effect can easily be explained considering that an appropriate adhesive force should be added to the right-hand-side of Eq. (4). Furthermore, we have checked that the results do not change introducing disorder in the substrate and in three dimensions, when the layers are confined between two rigid plates. Finally, we have checked that the reduction of friction occurs in the same frequency range in which diffusivity is strongly enhanced, in agreement with earlier results [7]. The results of these investigations will be discussed in more detail in a forthcoming publication.

The phase diagram displayed in Fig. 4(c) indicates the frictional behavior of the system under a steady vertical oscillation. There are cases, however, where the external vibration acts only for a small amount of time, such in the case of earthquakes or avalanches triggered by seismic waves. To address this issue, we analyze changes in the stick-slip pattern for small vertical vibration of finite du-

ration T_v and frequency ω , with $T_v \gg 1/\omega_0$. To avoid discontinuities, we switch the perturbation on the bottom plate smoothly: $Z_{\text{bot}} = Z_0 + f(t, T_v)A \sin(\omega_0 t)$, where $f(t, T_v) = \{\tanh(t/\tau) - \tanh[(t - T_v)/\tau]\}/2$, with $\tau \ll T_v$. If we chose T_v to be of the same order of magnitude as the stick time, we observe that the perturbation typically leads to small changes in the slip patterns. When $\omega_1 < \omega_0 < \omega_2$, however, the systems exhibit a large slip event (see Fig. 5). After the perturbation is removed the system recovers the original stick-slip behavior, without any long-range memory effects. Our results suggest that catastrophic events are more likely to be triggered when the perturbation lies in a definite frequency interval.

In conclusion, we have clarified the role of vibrations in the frictional sliding of a confined system. The general mechanism for friction suppression that we have uncovered is based on the reduction of the effective interface contacts produced by vibrations. Since the results depend only on the relation between inertial and dissipative forces, we expect them to be valid for a wide class of sliding systems, including granular media and nanoscale interfaces. Further work in this direction could be useful to optimize friction control in technological nanodevices and to design better strategies to forecast the triggering of instabilities in materials and geosystems.

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