



# Quantum communication with photon-number entangled states and realistic photodetection

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## ABSTRACT

We address the effects of realistic photodetection, with nonunit quantum efficiency and background noise (dark counts), on the performances of quantum communication schemes based on photon-number entangled states (PNES). We consider channels based on Gaussian twin-beam states (TWB) and non-Gaussian two-mode coherent states (TMC) and evaluate the channel capacity by optimizing the bit discrimination threshold. We found that TWB-based channels are more robust against noise than TMC-based ones and that this result is almost independent on the statistics of dark counts.

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## 1. Introduction

Quantum communication schemes are aimed to improve security [1] and capacity [2] of channels upon exploiting the specific features of the involved quantum mechanical systems. Indeed, quantum-enhanced key distribution (QKD) and communication schemes have been developed for single qubit [3] or entangled qubit pairs [4], and practically implemented using faint laser pulses or photon pairs from spontaneous parametric down-conversion. More recently, much attention has been devoted to investigate the use of continuous variable (CV) systems and protocols, in particular, using the sub-shot-noise fluctuations of photon-number difference of two correlated beams [5], the sub-shot-noise modulations [6] and the sub-shot-noise fluctuations of the photon numbers in each of the correlated modes [7] have been proposed. Although for these CV schemes the unconditional security proofs have not been obtained yet [8], they are of interest and deserve investigations mostly due to the potential gain in communication effectiveness.

In this Letter we consider schemes where information is encoded in the degrees of freedom of a correlated state shared by the two parties. In particular, we address binary communication channels based on photon-number entangled states (PNES) [9] as the two-mode coherently-correlated (TMC) or twin-beam (TWB) states. The communication protocol is based on photon-number correla-

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tions and realized upon choosing a shared threshold to convert the outcome of a joint photon-number measurement into a symbol from a discrete alphabet. Notice that, in principle, entanglement itself is not needed to establish this class of communication channels, which are based on photon-number correlations owned also by separable mixed states. On the other hand, purity of the support state is relevant to increase security of the channel, and the joint requirement of correlation and purity leads to individuate PNES as a suitable choice for building effective communication channels.

In this Letter we consider PNES-based quantum communication with realistic photodetection, which is affected both by nonunit quantum efficiency and background noise (dark counts), and analyze the effects of imperfections on the performances of channels. We consider channels based on TWB and TMC and evaluate the channel capacity by optimizing the bit discrimination threshold in the presence of noise. As we will see, TWB-based channels are more robust against noise than TMC-based one and this result is almost independent on the statistics of dark counts.

The Letter is structured as follows. At first we briefly introduce the two classes of PNES and illustrate the binary communication protocols. Then, we address noise in the decoding stage and derive the channel capacities in the presence of noise.

## 2. Binary communication with photon-number entangled states

PNES are bipartite states of two modes of the field with Schmidt decomposition in the Fock number states. They may be written as

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_n \psi_n |n, n\rangle, \quad (1)$$

where  $|n, n\rangle = |n\rangle_1 \otimes |n\rangle_2$  and  $N = \sum_n |\psi_n|^2$ . PNES can be generated by means of parametric processes, either in optical oscillators or amplifiers [10]. Generation of PNES have been reported with photon-number statistics varying from sub-Poisson [11] to super-Poisson [12–14]. As a matter of fact, several quantum communication schemes and QKD protocols have been proposed exploiting PNES correlations with coding based on the beams intensity [6,15] or intensity difference [16]. In particular, the degenerate pair-coherent states [17,18], also referred to as two-mode coherently correlated (TMC) [7] have been suggested as an effective channel. TMC may be written in the Fock basis as follows

$$|\lambda\rangle = \frac{1}{\sqrt{I_0(2|\lambda|)}} \sum_n \frac{\lambda^n}{n!} |n, n\rangle. \quad (2)$$

The peculiarity of TMC is that they show sub-Poisson statistics for each of the beam. The corresponding QKD scheme is based on the fact that due to the strong intensity correlations one may decode a random bit sequence which will be correlated for the two remote sides carrying out independent but simultaneous intensity measurements on each of the two spatial twin modes. The realistic security of the scheme is based on the checking the beam statistics against the expected one corresponding to the fixed known state parameter. It was shown that realistic eavesdropping attempts cause statistics degradation to super-Poisson distribution and introduce perturbations in the obtained density matrix which are significant enough to be detected thus making eavesdropping ineffective [7].

The bits decoding for PNES-based communication protocol is quite natural—each of the legitimate users measure the incoming photon number for a next time slot and compare the obtained value to a given bit threshold. If the current photon-number value is above the threshold the corresponding bit value is considered to be equal to 1, while if the photon number is below the threshold the bit value is equal to 0:

$$B = \begin{cases} n \leq T \rightarrow 0, \\ n > T \rightarrow 1. \end{cases} \quad (3)$$

The threshold may be optimized, or set to a predetermined value, e.g. the integer part of the mean photon number. With the latter choice the alphabet extension to the 4- and 8-letter sets was shown to increase the information capacity and make the protocol secure against intercept-resend attacks [15].

Another relevant class of the PNES states (1) is the twin-beam state (TWB). The Fock expansion is given by

$$|x\rangle = \sqrt{(1-x^2)} \sum_n x^n |n, n\rangle. \quad (4)$$

TWB are Gaussian states [19,20] and the photon statistics of the two modes is super-Poisson in contrast to the sub-Poisson statistics of the TMC state modes. The TWB states can be used for the implementation of the same quantum communication protocol (3) exploiting the photon-number CV information coding, though the security issues remain an open question (especially the security against the intercept-resend eavesdropping, which is based on the sub-Poisson statistics check for the TMC). The TWB-based protocol may possibly require the use of the additional degrees of freedom in order to force the eavesdropper to guess the measurement and generation bases like it is in the celebrated BB84 single-qubit QKD protocol [3].

The practical implementation of PNES-based communication protocols (3), either based on TMC or TWB, crucially depends on the influence of the realistic lossy optical media and realistic noisy photodetectors. Those are the main sources of noise for the secure QKD protocols implementation since they restrict the communication distances and rates. The influence of losses on the perfor-

mance of the PNES-based channels (in comparison to classically mixed states) has been previously investigated [9]. Here we focus on the effect of the nonideal photodetection.

### 3. Noisy photon-counting

In order to investigate the effect of the noisy photon-counting on PNES-based quantum channel we model the detector as an ideal one preceded by a beam-splitter of transmittivity  $\eta$  equal to the detection losses. The first port of the beam-splitter is fed by the signal state, while the other port is excited by an auxiliary state that reproduces the background noise. The action of the beam-splitter is described by the operator  $U_\phi = \exp\{\phi(a_1^\dagger a_2 - a_1 a_2^\dagger)\}$ , which in the Heisenberg picture corresponds to the modes evolution [21]:

$$U_\phi^\dagger \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} U_\phi = \mathbf{B}_\phi \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{B}_\phi = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}, \quad (5)$$

where  $\eta = \cos^2 \phi$ . The evolution of the Fock number basis may be expressed as

$$U_\phi |n_1\rangle \otimes |n_2\rangle = \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} A_{k_1 k_2}^{n_1 n_2} |k_1 + k_2\rangle \otimes |n_1 + n_2 - k_1 - k_2\rangle, \quad (6)$$

where the transfer matrix  $A_{k_1 k_2}^{n_1 n_2}$  is given by

$$A_{k_1 k_2}^{n_1 n_2} = \sqrt{\frac{(k_1 + k_2)! (n_1 + n_2 - k_1 - k_2)!}{n_1! n_2!}} (-1)^{k_2} \times \binom{n_1}{k_1} \binom{n_2}{k_2} \sin \phi^{n_1 - k_1 + k_2} \cos \phi^{n_2 + k_1 - k_2}. \quad (7)$$

Upon writing the signal as  $\rho = \sum_{nm} \rho_{nm} |n\rangle \langle m|$  and the noise state as  $\nu = \sum_p \nu_p |p\rangle \langle p|$ , then the probability to have  $s$  counts at the output is given by

$$p_s = \text{Tr}[U_\phi \rho \otimes \nu^\dagger U_\phi |s\rangle \langle s| \otimes \mathbb{I}], \quad (8)$$

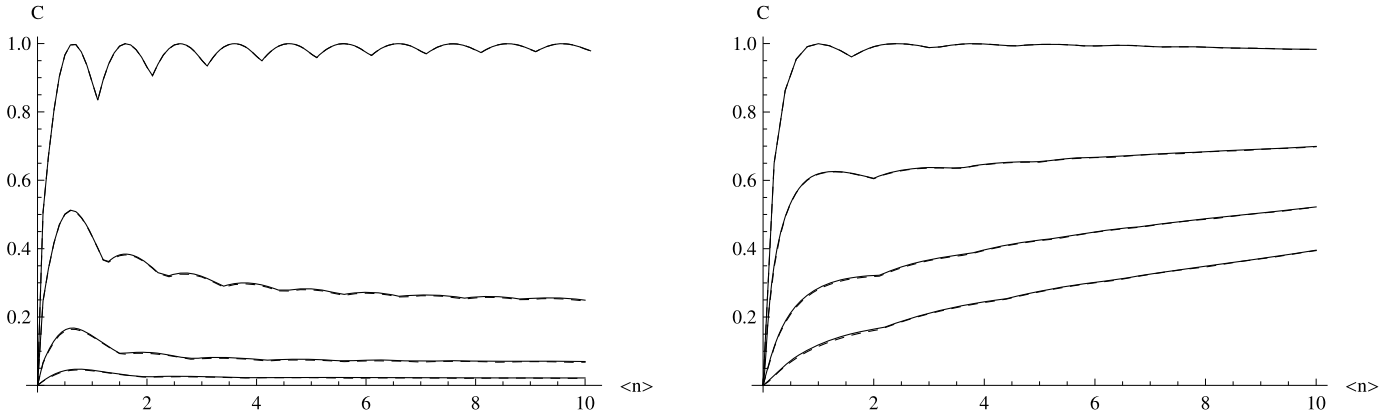
which may be written as

$$p_s = \sum_{n=0}^{\infty} \rho_{nn} \left[ \sum_{p=0}^{\infty} \nu_p \left( \sum_{k=0}^s A_{k, s-k}^{n, p} \right)^2 \theta(n + p - s) \right] = \frac{1 - \eta^s}{\eta} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} (1 - \eta)^n \rho_{nn} \eta^p \nu_p \binom{n + p - s}{p - s} \binom{p}{s} \times {}_2F_1 \left( -n, -s, 1 + p - s, -\frac{\eta}{1 - \eta} \right)^2 \theta(n + p - s), \quad (9)$$

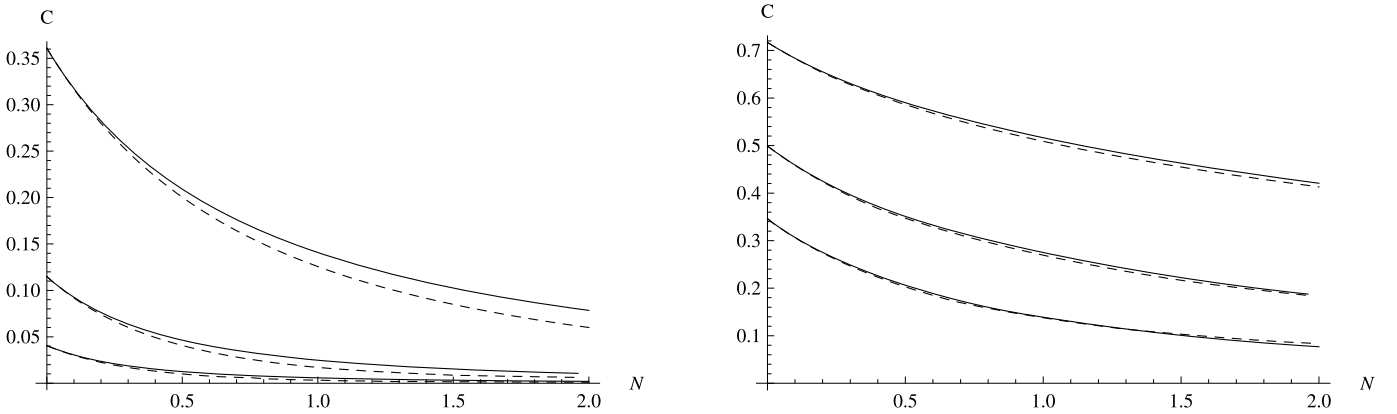
$\theta(x)$  being the Heaviside step function. Eqs. (8) and (9) express the relation between the probability of photocounts in a noisy detector and the actual photon distribution of the incoming signal. Once the joint probability distribution is known we may evaluate the mutual information between the two parties and optimize it against the threshold. For the binary coding of Eq. (3) the two parties infer the same symbol with probabilities

$$p_{00} = \sum_{p=0}^T \sum_{q=0}^T P_\eta(p, q), \quad p_{11} = \sum_{p=T}^{\infty} \sum_{q=T}^{\infty} P_\eta(p, q). \quad (10)$$

In the ideal case, i.e. with no losses, PNES-based protocols achieve  $p_{00} + p_{11} = 1$ , due to perfect correlations between the two modes. On the other hand, if  $\eta \neq 1$  the unwanted inference events “01” and “10” may occur with probabilities



**Fig. 1.** Channel capacity (mutual information maximized over the bit threshold) for the TMC- (left) and TWB-based quantum channels as a function of their average number of photons for different values of the detection efficiency  $\eta$ ; from bottom to top:  $\eta = 0.5$ ,  $\eta = 0.7$ ,  $\eta = 0.9$  and  $\eta = 1$ , the latter corresponding to the ideal detectors. The detection noise is  $N = 0.2$ , noise statistics is either thermal (solid lines) or Poisson (dashed lines), almost coinciding.



**Fig. 2.** Channel capacity (mutual information maximized over the bit threshold) for the TMC- (left) and TWB-based quantum channels as a function of the detector noise average number of photons for different values of the loss parameter  $\eta$ ; from bottom to top:  $\eta = 0.5$ ,  $\eta = 0.7$ ,  $\eta = 0.9$ , average number of photons in a signal mode is  $\langle n \rangle = 5$ , noise statistics is thermal (solid lines) or Poisson (dashed lines).

$$p_{01} = \sum_{p=0}^T \sum_{q=T}^{\infty} P_{\eta}(p, q), \quad p_{10} = \sum_{p=T}^{\infty} \sum_{q=0}^T P_{\eta}(p, q). \quad (11)$$

The probabilities are not independent since the normalization condition  $p_{00} + p_{10} + p_{01} + p_{11} = 1$  holds. The mutual information between the two alphabets reads as follows

$$I_2 = \sum_{i=0}^1 \sum_{j=0}^1 p_{ij} \log \frac{p_{ij}}{q_i r_j}, \quad (12)$$

where

$$q_i = p_{i0} + p_{i1}, \quad i = 0, 1, \quad r_j = p_{0j} + p_{1j}, \quad j = 0, 1, \quad (13)$$

represent the marginal probabilities, i.e. the unconditional probabilities of inferring the symbol “ $i$ ” (“ $j$ ”) for the first (second) party. The mutual information, once the average number of input photons and the loss parameter have been set, depends only on the threshold value  $T$ . The channel capacity  $C = \max_T I_2$  corresponds to the maximum of the mutual information over the threshold. We have obtained the channel capacity numerically by looking for the optimal bit discrimination threshold as a function of the input energy and of the intensity of the noise states, assuming that detectors are equivalent for the both modes. In our calculations we considered the background noise either with Poisson statistics so that  $\nu_p = e^{-N} \frac{N^p}{p!}$ , or thermal statistics  $\nu_p = \frac{N^p}{(N+1)^{p+1}}$ , where  $N$  is the average number of photons of a noise mode.

The channel capacities for both TMC and TWB states versus a signal mode intensity are shown in Fig. 1 for noise average photon number  $N = 0.2$  and various detector efficiencies  $\eta$ . At fixed energy the channel capacity is larger for TWB than for TMC, even when being reduced by an inefficient detection. The channel capacities were also calculated for the fixed signal energy with respect to increasing noise intensity. The results are given at Fig. 2, the signal mode average photon number is  $\langle n \rangle = 5$ . Again, the TWB states are more effective than TMC at the same energy and appear to be more robust against the detection noise. Also, the results are almost indistinguishable for Poisson and thermal noise, the latter being a bit more destructive, especially for TMC states for higher noise intensities.

#### 4. Conclusions

We have analyzed the effect of detection noise, quantum efficiency and dark counts, on the performance of PNEs-based quantum channels. Our results show that the TWB-based channels are more robust against noise. The statistical properties of noise do not play a significant role, as the noise impact mostly depends on its intensity.

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