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Multiplex Networks: A Framework for Studying Multiprocess Multiscale Connectivity Via Coupled-Network Theory With an Application to River Deltas

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Key Points:
- We present a general framework based on coupled network theory to study multiprocess connectivity in Earth Sciences
- Multiplex networks allow to simultaneously account for within- and across-process interactions
- We reveal emergent transport properties in river deltas as a function of the coupling between channels and islands

Supporting Information:
- Text S1

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Abstract Transport of water, nutrients, or energy fluxes in many natural or coupled human natural systems occurs along different pathways that often have a wide range of transport timescales and might exchange fluxes with each other dynamically. Although network approaches have been proposed for studying connectivity and transport properties on single-layer networks, theories considering interacting networks are lacking. We present a general framework for transport on multiscale coupled-connectivity systems, via multilayer networks which conceptualize the system as a set of interacting networks, each arranged in a separate layer, and with interactions across layers acknowledged by interlayer links. We illustrate this framework by examining transport in river deltas as a dynamic interaction of flow within river channels and overland flow on the islands, when controlled by the flooding level. We show the potential of the framework to answer quantitative questions related to the characteristic timescale of response in the system.

Plain Language Summary The physical processes that shape landscapes leave behind patterns of connectivity along which fluxes occur via a multitude of processes, for example, flow through channels, subsurface or overland flow. The connectivity imposed by those processes (e.g., channel networks) exerts a significant control on the evolution and form of the underlying systems. We introduce a framework based on coupled networks, Multiplex, that allows to quantify the connectivity properties emerging from the simultaneous action of different processes, enabling thus to assess the overall system properties and dynamics. We illustrate this framework by examining the case of river deltas, where intermittent flooding and exchange of water, sediment, and nutrients between the channels and the islands maintains the delta top by trapping sediment, stabilizing banks, and enriching rivers with carbon and nutrients. By describing the delta system as a Multiplex—integrating the connectivity imposed by confined (in the channel network) and overland (on the islands) flows as well as the interactions (flux exchange) between them—we show the emergence of system transport properties and dynamics not foreseen by analyzing each process separately, and therefore revealing key information essential to predict the system response under changing forcing.

1. Introduction

Conceptualizing connectivity within a graph theoretic framework for studying processes on the Earth's surface has seen increased interest over the last decades (e.g., see reviews by Phillips et al., 2015; Heckmann et al., 2015; Passalacqua, 2017, and references within). In a graph or network, nodes represent physical locations or state variables, and links represent the direction and strength of connectivity or interaction between nodes. In many physical systems, transport takes place by more than one mechanism, for example, overland flow and channel flow, and over different transport pathways, for example, within the channel network and/or within the interdispersed set of islands as overland flow, with exchange occurring between these two interconnected systems depending on the magnitude of the system forcing and possibly local conditions. To represent such a process within a network framework requires conceptualizing it as a system of distinct networks, each with different transport properties, and with interaction allowed between networks to accommodate the flux exchange. It is expected that considering the overall system connectivity in this enlarged network perspective will result in emergent transport properties and dynamics not possible to decipher by analyzing each network separately, and therefore revealing key information essential to predict the system response under changing forcing.

In recent years, a new framework that generalizes the traditional representation of networks to the so-called multilayer networks was introduced (Boccaletti et al., 2014; De Domenico et al., 2013; Kivelä et al., 2014; Mucha et al., 2010). A multilayer network represents the different connectivities arising from various processes as distinct networks (layers) but allows at the same time to represent interactions between separate layers by introducing interlayer links (across process interactions). The application of this framework has spanned diverse disciplines ranging from social networks (e.g., propagation of information across different social media platforms; Cozzo et al., 2013) to transportation networks (e.g., transportation in a multiplatform system—subway and bus; De Domenico et al., 2014) and biochemical networks (e.g., spreading of two diseases, which interact cooperatively; Sanz et al., 2014), to name a few. Despite the enormous potential for the application of this framework to the study of diverse surface or subsurface processes, to the best of our knowledge, it has not been utilized yet in the earth sciences community. One of the reasons is that the theory of multilayer networks has been mostly developed for undirected networks, that is, when the flow in each of the layers is not restricted to a specific direction (e.g., information flow in social networks). However, most of the flow networks in hydrology and geomorphology have distinct directionality imposed by topographic, hydraulic, or geologic gradients, which call for the need to extend the theory so that it integrates the directionality of such systems into the current framework. This theory was recently proposed by Tejedor et al. (2018), who provided analytical proofs and numerical evidence that in directed multiplex, intermediate values of coupling between layers can accelerate the overall transport rates in the system and reduce the timescale needed to reach an equilibrium state. The implications of this result for real systems are important as the interplay between the timescales to equilibrium and timescales associated using system perturbation can result in unexpected long-term system states.

The aim of this paper is to show, in a simple and physically intuitive way, the framework of directed multilayer networks to study geophysical problems. To this end, we examine a river delta, as an example of a complex interconnected system, where two transport mechanisms—channelized flow within the channel network and overland flow on the islands—with very different timescales contribute significantly to the overall system transport dynamics (Hiatt & Passalacqua, 2015). The flux exchange between these two transport mechanisms depends on variables such as river discharge, and therefore, it is interesting to ask under what conditions and through which local interactions (exchanges) the overall system might exhibit accelerated transport not expected by each system alone. Quantifying the system's response timescale as a function of the level of discharge (coupling) and comparing it with the timescale of the forcings (e.g., time during which certain discharge level is exceeded) can reveal important information about relevant biogeomorphic processes on deltas, for example, sediment trapping and delivery of nutrients to the delta top promoting, by complex feedbacks, the development of vegetation (e.g., Larsen & Harvey, 2010; Nardin & Edmonds, 2014), and nutrient processes (e.g., Hiatt et al., 2018).

2. The Mathematical Framework: From Single-Layer to Multilayer Connectivity

Specifying the connectivity of traditional single-layer networks (referred to herein as monoplex) only requires two indices per link (parent and child node), making matrices a suitable representation of networks. Thus, the connectivity structure of a graph consisting of N nodes interconnected by links can be uniquely specified by a square $N \times N$ matrix called Adjacency matrix (A), whose entry a_{uv} is a nonnegative number (strength of the connection) if there exists a (vu) link from node v to node u, and 0 otherwise. For multilayer networks, two indices are not enough, since it is also necessary to specify the layers to which each of the two nodes connected by a link belongs. Tensors are the natural generalization of matrices when a higher dimensionality is required. Consequently, we define the multilayer Adjacency tensor M whose entry $M_{uv}^{\alpha\beta} > 0$ denotes a link starting at node v at layer β and ending on node u in layer α (De Domenico et al., 2013).

There is a specific subclass of multilayer networks called multiplex networks (hereafter referred to as multiplex; De Domenico et al., 2013; Gómez et al., 2013), wherein each layer consists of the same set of nodes but possibly different topologies (set of links) and the layers interact with each other only via the

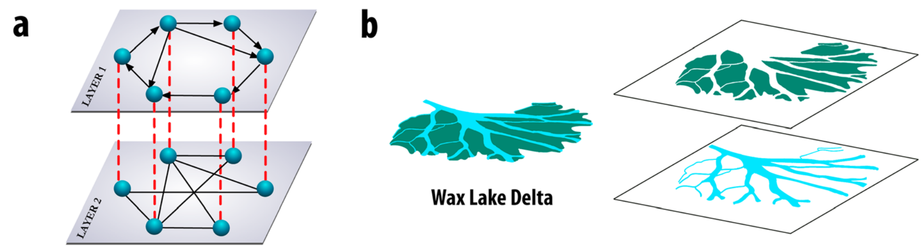

Figure 1. Delta multiplex (a) *Illustration of a multiplex:* Multiplex are coupled multilayer networks where each layer consists of the same set of nodes but possibly different topologies (set of links), and layers interact with each other only via replica nodes in each layer (dashed lines) (b) *Wax Lake Delta Multiplex*. Illustration of the Wax Lake delta in the Louisiana coast (United States). The delta multiplex consists of two layers: Layer 1 (bottom) accounts for the channel connectivity and layer 2 (top) represents the connectivity that arises from overland flow on islands. For more details about the multiplex structure see supporting information.

counterpart nodes in each layer (Figure 1a). We are especially interested in the multiplex because (1) they are relevant to networks that are embedded in space, where interactions across layers are not expected to happen between distant nodes but only between counterpart nodes in the different layers (e.g., in deltas, the exchange of fluxes between channels and islands occurs locally); and (2) the limitation of having the interlayer connectivity only among counterpart nodes makes the mathematical representation of multiplex simpler allowing us to project the Adjacency tensor in an $NP \times NP$ *matrix*, called *supra-Adjacency* matrix, $\mathscr{A}$, where N is the number of nodes per layer, and P is the number of layers. For the case of two layers, $\mathscr{A}$ takes the following form:

$$\mathscr{A} = \begin{pmatrix} A^{(1)} & I \\ I & A^{(2)} \end{pmatrix}, \tag{1}$$

where I is the $N \times N$ identity matrix. Note that in $\mathscr{A}$ replica nodes are labeled to satisfy $u + kN$ for $k = 0,1, \dots P\text{-}1$. Hence, the *supra-Adjacency* matrix is a block matrix, where each of the diagonal blocks encodes the intralayer connectivity of the respective layers and the interlayer connectivity between replica nodes is represented by the identity matrices located in the off-diagonal blocks.

Another important operator in graph theory is the *Laplacian* L, which is the cornerstone of spectral graph theory. In the case of a monoplex, L is a matrix that can be solely derived from the Adjacency matrix. It is defined as $L{=}S - A$, where S is the $N \times N$ diagonal matrix with diagonal entries $s_{vv} = \sum_{u=1}^{N} a_{uv}$, that is, the sum of the weights of all the links leaving node v (Note that we denote here by L what is generally known as the out-Laplacian; Tejedor et al., 2015a). Equivalently, a *supra-Laplacian* matrix $\mathscr{L}$ can be defined for any multiplex. For the case of two layers, $\mathscr{L}$ is defined as (Gómez et al., 2013; Tejedor et al., 2018):

$$\mathscr{L} = \begin{pmatrix} D_1 L^{(1)} + D_X I & -D_X I \\ -D_X I & D_2 L^{(2)} + D_X I \end{pmatrix}, \tag{2}$$

where D_1 and D_2 are intralayer diffusion coefficient of layer 1 and 2, respectively, D_X is the interlayer diffusion coefficient, and $L^{(1)}$ and $L^{(2)}$ are the Laplacian operators of the intralayer connectivity of the respective layers as defined for the monoplex. The nomenclature of the parameters D_1, D_2, and D_X as diffusion coefficients is reminiscent of the interpretation of L as the diffusive operator in networks (Newman, 2010). In a more general setting, we can interpret those coefficients as scalars that allow modifying the relative celerity of the process of each layer and the interlayer processes.

3. A Continuous Time Markov Chain as Proxy for Flux Dynamics in River Deltas

We use a simple Continuous Time Markov Chain (CTMC) model to approximate the dynamics and relative timescales for achieving steady state distributions when different values of coupling (flux exchange) are assumed between the channel and island layers. The CTMC relies on several assumptions such as (i) a constant rate of transition, that is, the partition of fluxes at a given bifurcation remains constant and proportional to the physical parameters of the network, for example, channel width; and (ii) the Markovian property, that is, the downstream direction that a given *package* of water or sediment particles takes at a given bifurcation depends only on the physical properties of that bifurcation and not on the trajectory of the package in its journey from upstream. Despite these assumptions, CTMC offers a good first-order approximation of the dynamics of the system.

The negative *supra-Laplacian*—$\mathscr{L}$ (see equation (2)) can be interpreted as the transition rate matrix of a CTMC (Masuda et al., 2017; Norris, 1997). The dynamics of the corresponding Continuous Time Random Walk (CTRW) are governed by

$$\dot{\boldsymbol{p}}(t) = -\mathscr{L}\boldsymbol{p}(t), \tag{3}$$

where the *i*th component of $\boldsymbol{p}(t)$ represents the probability that the CTRW visits node i at time t.

If the directed network is strongly connected, a unique stationary distribution of probability $\boldsymbol{p_s}$, referred in the rest of the paper as steady state, exists (see Tejedor et al., 2018, for further details) such as

$$\mathscr{L}\boldsymbol{p_s} = 0. \tag{4}$$

The rate of convergence toward the steady state given by $\boldsymbol{p_s}$ is exponential (asymptotically) with rate $\mathrm{Re}(\Lambda_2)$, where Λ_2 is the eigenvalue with the smallest nonzero real part (Lodato et al., 2007; Masuda et al., 2017; Tejedor et al., 2018). Equivalently, the timescale of convergence to steady state, τ, is inversely proportional to the rate of convergence $(\tau \propto 1/\mathrm{Re}(\Lambda_2))$. Note that the spectrum of eigenvalues of $\mathscr{L}$ is in general complex since it is not symmetric. Considering the definition of $\mathscr{L}$ (see equation (2)), its eigenvalue spectra, and more specifically $\mathrm{Re}(\Lambda_2)$, depend on the following: (1) the topology of the connectivity of layer 1, L_1, (2) the diffusion coefficient of layer 1, D_1, (3) the topology of the connectivity of layer 2, L_2, (4) the diffusion coefficient of layer 2, D_2, and (5) the interlayer diffusion coefficient, D_X.

4. River Deltas as Multiplex Networks

River deltas are depositional landforms forming at the mouths of major rivers when sediment-laden water slows down as it enters a body of standing water (Gilbert, 1885). Deltas contain nutrient-rich sediments that support agriculture, their deposits are often rich in oil and hydrocarbons, and provide a variety of environmental services (Barbier et al., 2011; Brondizio et al., 2016; Cohen et al., 1997; Hoanh et al., 2010; Woodroffe et al., 2006). However, many major deltas are losing land because of the combined effects of (i) sediment deprivation due to dams and levees construction, (ii) accelerated subsidence due to soil compaction exacerbated by groundwater and/or oil extraction, and (iii) rising sea levels (Blum & Roberts, 2009; Ericson et al., 2006; Giosan et al., 2014; Syvitski et al., 2005, 2009). Deltas consist of a network of channels that tiles their surface and that are surrounded by islands (interchannel areas) regularly inundated by river flooding and tides. A substantial progress in understanding deltaic systems can be made by studying the structure and function of their channel networks (Morisawa, 1985; Smart & Moruzzi, 1971; Tejedor et al., 2015a, 2015b, 2016, 2017). However, it is well known that water and sediment fluxes are not only confined to the delta channel network. There are event-related, seasonal, and permanent (e.g., close to the delta shoreline) water and sediment exchanges between channels and islands. Passalacqua (2017) described this exchange of fluxes between channels and islands in deltas as a "leaky network" of channels and islands. The simultaneous action of two coupled mechanisms of transport, each one with different topologies and transport properties (channelized flow in the channel network and overland flow on islands), makes deltas a prototype example that can be described by Multiplex networks.

4.1. Wax Lake Delta as a Case Study

We use the Wax Lake Delta as a case study to show the potential of the framework for a real topology where field measurements of transport rates are available both for channelized and overland flow. The Wax Lake delta is a river-dominated delta located in coastal Louisiana, United States. Subaerial land developed after the 1970s flood, and the delta has been rapidly prograding ever since (Paola et al., 2011; Roberts et al., 1997; Shaw et al., 2013). Lidar surveys have shown that 83% of the delta top experienced aggradation between 2009 and 2013 (Wagner et al., 2017). Primary channels transport water and sediment in the delta to the Atchafalaya Bay, and secondary channels connect the delta channel network to the island interiors (Shaw et al., 2013).

Using the channel network connectivity of Wax Lake delta—channel layer (Layer 1—denoted here and after as layer C)—together with the island connectivity—island layer (Layer 2—denoted here and after as layer I)—(see Figure 1b and supporting information for further details about the connectivity used and a brief discussion of the deltaic system), we examine the timescale of response of this coupled system. Without loss of generality, we have set the value of $D_I = 1$. The value of $D_C = 7$ has been selected in order to generate a timescale of transport on channels that is 3 times faster than that of the islands, which is compatible with data collected from field campaigns (see Hiatt & Passalacqua, 2015—channels ~4.4 hr; islands ~14.3 hr). The rate of water and sediment exchange between the channels and islands is controlled by hydrologic (e.g., level of water discharge) and ecogeomorphic (e.g., vegetation, existence of secondary channels connecting the channel network to the interior of the islands) attributes. The effect of vegetation is summarized into the value of D_I, that is, more vegetated islands exhibit a higher roughness and therefore are expected to have a lower value of the diffusion coefficient D_I. Thus, the value of D_X is mostly controlled by the discharge level, as here other forcings such as tides and wind are ignored. Note that we assume that the value of D_X is homogeneous across the delta. This assumption is an oversimplification given the existence of secondary channels in some of the islands, gradients in vegetation, and connectivity toward the distal part of the deltaic system, etc., and therefore, a spatially explicit modulation of this parameter would make the model more realistic. However, for the sake of simplicity in the presentation of the framework, we assume uniform values of D_X, showing that even in this simplified scenario, interesting and unexpected system-wide behaviors emerge from the coupled dynamics. This simplification also allows us to demonstrate that the system response described below does not emerge from heterogeneity in the spatial patterns of D_X, but it is intrinsic to the coupled connectivity between the channel and island layers. In a more general setting, the parametrization of the diffusion coefficients (D_C, D_I, D_X) would require identifying the main variables controlling the values of the diffusion coefficients. New approaches based on information theory to unveil nonlinear dependencies among variables at different temporal scales constitute a powerful toolbox for this purpose (e.g., Goodwell & Kumar, 2017; Ruddell & Kumar, 2009; Sendrowski & Passalacqua, 2017).

As detailed in section 3, given a multiplex topology and the set of parameters (D_C, D_I, D_X), we can compute the timescale of convergence to steady state transport as $\tau \propto 1/\mathrm{Re}(\Lambda_2)$. In the delta multiplex example, when the input boundary condition (discharge at the apex) is changed, a redistribution of water, solids, and solutes occurs in the channel-island complex. Thus, if the new boundary condition were to be kept constant, a steady state distribution of water, sediment, nutrients, and transport rates would be eventually achieved everywhere in the delta top. For deltas, as for many other natural systems, boundary conditions such as incoming discharge vary over time. Furthermore, the characteristic time (e.g., time elapsed while a certain level of discharge is exceeded) associated with that variability also involves multiple scales. For instance, we can imagine two different scenarios where an incoming level of discharge Q is exceeded due to (i) seasonal variability (e.g., high flow conditions in winter vs low flow conditions in summer) or (ii) an extreme event (e.g., storm). The characteristic timescales of forcings together with the timescale of response would dictate whether a significant reorganization of sediment and nutrients can take place, or even a new steady state could be reached. Precisely for these reasons, we do not assume in our analysis that a steady state distribution is achieved, and therefore, we do not focus our analysis on the actual steady state distribution that would emerge for a given boundary condition if maintained constant. Our analysis capitalizes on the potential of the multiplex framework to estimate the overall system timescale involved in achieving the steady state. This timescale is very relevant to diverse biogeomorphic processes because it provides the magnitude of the characteristic time to achieve a substantial redistribution of sediment and nutrients on the delta top when the boundary conditions are changed (e.g., flooding). Thus, we can interpret the timescale τ as the *characteristic response time* of the delta to varying discharges.

By analyzing the behavior of τ as a function of the interlayer coupling, D_X, (Figure 2a) the existence of four regimes stands out: (1) *Linear:* The dynamics in the channel network and on the islands are effectively decoupled wherein the rate of flux exchange between both layers (D_X) is the limiting factor (note that the bankfull condition does not have to be exceeded to observe flow into the islands; e.g., Hiatt & Passalacqua,

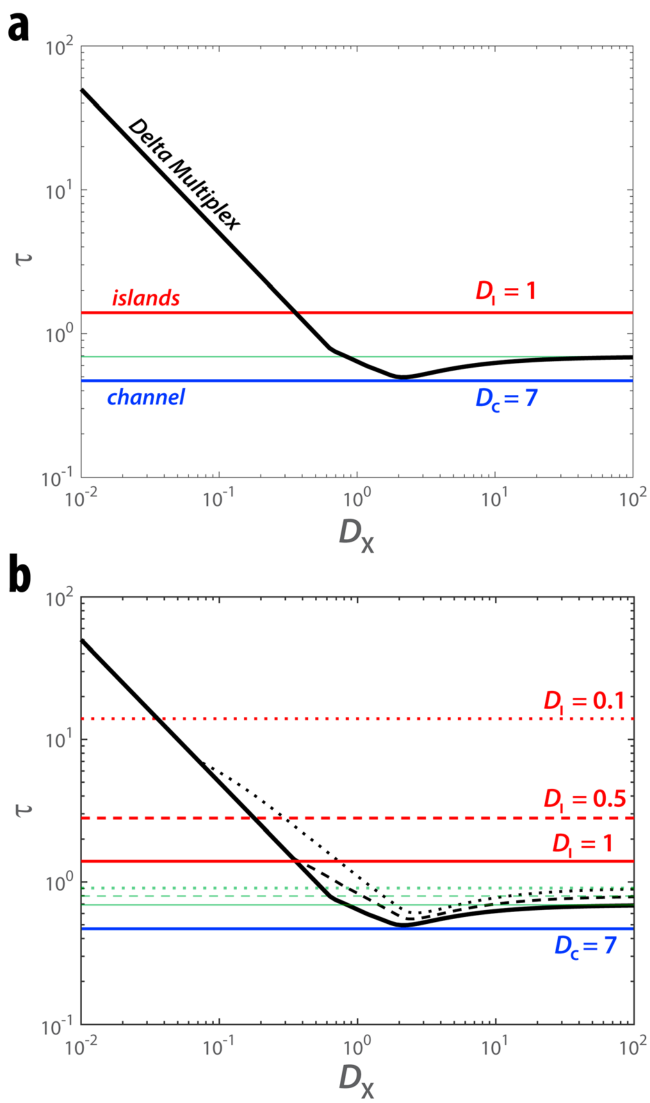

Figure 2. Flux dynamics on the Wax Lake Delta multiplex. We show for the Wax Lake multiplex, the timescale of convergence to steady state, τ, as a function of interlayer coupling, D_X, which is mostly controlled by water discharge. Panel (a) shows the emergence of a nonmonotonic behavior of τ as function of D_X, when the values of diffusivity of each layer are set to (D_C, D_I) = (7, 1) to reproduce the ratio of transport timescales channel to island reported from field campaigns. Panel (b) shows the effect of island roughness (e.g., vegetation) in the response timescale of the delta multiplex. For intermediate values of D_X, the timescale of the delta multiplex τ increases for higher island roughness—D_I = 1 (solid lines), 0.5 (dashed lines), and 0.1 (dotted lines)—reducing effectively the coupling between the channels and islands for the same values of D_X.

2015). In this regime, the timescale of convergence to steady state (τ) decreases linearly as $2D_X$. (2) *Sublinear*: The coupling between channels and islands starts to be more significant but is limited by the slower diffusion process in the islands. Here an increase in D_X, that is, a larger water discharge, translates into a sublinear decrease ($\tau \propto D_X^{-\alpha},\ \ \alpha < 1$) in the timescale of convergence to steady state for the overall delta. (3) *Asymptotic*: For very large values of discharge (i.e., $D_X \gg \mathrm{Re}\left(\lambda_2^I\right),\ \mathrm{Re}\left(\lambda_2^C\right)$) the two layers are completely coupled. This scenario can occur when the water discharge is large enough to generate sheet flow on the whole system, where the counterpart nodes in the different islands and channels are fully synchronized, behaving as single nodes. (4) *Prime*: This regime, characteristic of multiplex with directed connectivity in at least one of its layers (Tejedor et al., 2018), occurs for intermediate values of coupling (discharge, D_X), wherein the rate of convergence in the overall system achieves the shortest timescale, even shorter than in the asymptotic regime. In this scenario, both islands and channels contribute significantly to the total transport but conserving a relative degree of independence in their internal dynamics (i.e., not fully synchronized or decoupled). Physically, this coupling regime can be interpreted as levels of discharge that produce rates of channel-island flux exchange similar to the rates characteristic of channel transport $\left(D_X \sim \mathrm{Re}\left(\lambda_2^c\right)\right)$. Thus, islands although characterized by slower rates of transport, if provided with the right amount of flux (given the ratio of channel and island transport rates), are able to facilitate the acceleration of the overall rate of transport alleviating bottleneck scenarios at the channels. Thus, there exists a *sweet spot* for certain values of discharge (optimal coupling) for which the timescale τ achieves its minimum, and either an increase or decrease of discharge with respect to that optimal value would translate in an increase of τ (decrease in the transport rate).

It is important to notice that although the parameter that controls the flux exchange between the channel and island layers, D_X, is solely interpreted in terms of water discharge, island roughness (e.g., due to vegetation) has been shown to effectively play a fundamental role in the water exchange between islands and channels (Hiatt & Passalacqua, 2017). The multiplex framework allows us to easily assess the effect of different island roughness (mediated by the value of D_I) in the system-wide response. When different scenarios of increasing island roughness (D_I = 1, 0.5 and 0.1) are explored for a constant D_C = 7 (Figure 2b), the transition from the linear to the sublinear regime shifts to smaller values of D_X (discharge), acknowledging the fact that the increased island roughness (i.e., lower value of D_I) makes the transport on islands the limiting factor for a larger range of discharges. The immediate consequence of this shift in the transition from linear to sublinear is that the system-wide response is significantly slowed down for the same values of discharge under increased roughness scenarios. Finally, it is also interesting to note the shift in the position of the minimum timescale of convergence to steady state, τ, toward higher values of discharge (D_X) when higher values of island roughness (lower values of D_I) are explored. Thus, for example, to achieve the shortest timescale of response of the system in high roughness scenarios (D_I = 0.1), a 20% increase in the discharge (D_X) is necessary when compared to the case of low roughness (D_I = 1).

Depending on the discharge levels (for given values of D_I and D_C), the delta multiplex behaves as a channel-dominated system or a coupled channel-island complex. The geomorphic (e.g., island aggradation) and biogeochemical (e.g., vegetation types, nutrient nourishment and nitrogen fixation) consequences of

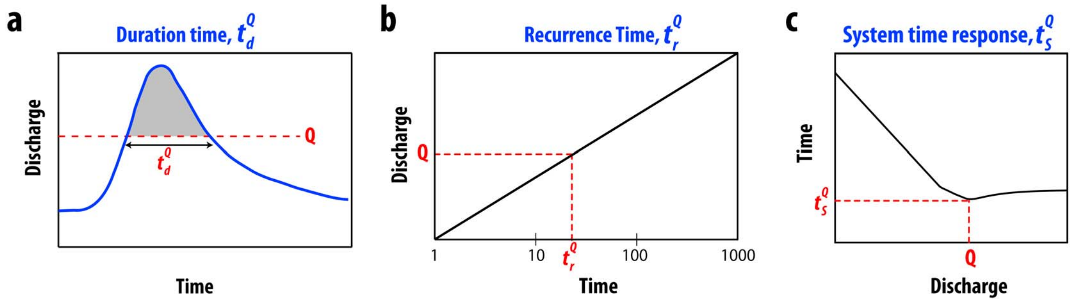

Figure 3. The timescales associated with discharge Q. (a) t_d^Q is the timescale associated with the duration of the forcing, that is, the time during which the value of Q is exceeded; (b) t_r^Q is the time of recurrence of the forcing of magnitude Q; and (c) t_S^Q is the timescale of response of the channel-island delta system (see Figure 2) when both layers are coupled by a discharge level Q.

operating in one or the other scenario are apparent. However, to fully evaluate the overall system behavior, there are three relevant timescales associated with a discharge Q (Figure 3) that should be taken into consideration: t_d^Q is the timescale associated with the duration of the forcing, that is, the time during which the value of Q is exceeded; t_r^Q is the time of recurrence of the forcing of magnitude Q; and t_S^Q is the timescale of response of the channel-island delta system when both layers are coupled by a discharge level Q. The multiplex framework allows to put into perspective these three timescales. Thus, the delta as a whole would be efficient in redistributing sediments and nutrients, (i.e., it behaves as a channel-island complex), if $t_S^Q \leq t_d^Q$, that is, the time of response of the system is comparable with the duration of the forcing. Thus, a deltaic system is resilient, that is, it exhibits aggradation rates that are fast enough to self-maintain the delta and the ecosystem services that it provides, if its overall delta connectivity has evolved to a state wherein the prime regime of transport: (1) emerges for water discharge Q (interlayer coupling) values with a recurrence time t_r^Q that is short enough to allow periodic redistribution of fluxes at the delta scale and (2) characterized by small values of t_S^Q (i.e., comparable in magnitude with the timescale of the transport in the channels).

5. Conclusions and Perspectives

To investigate transport properties of multiprocess multiscale connected systems, we introduce the framework of multilayer networks which allows to quantify properties of the system as a whole, not accessible by studying each system separately. We illustrate this framework by examining the flux dynamics in a river delta system, where channelized (within the channel network) and overland (on the islands) flows are considered. We represent the delta system as a two-layer multiplex, wherein each layer consists of the same number of nodes, but the connectivity among them is different and representative of each process. The degree of coupling among layers denotes the flux exchange in between the two transport processes, and in this study is driven by the discharge level, although a strong control is also exerted by the relative roughness of the islands (e.g., vegetation). To illustrate the potential of this framework, we investigate the timescale of convergence to the steady state flux distribution for different degrees of coupling, revealing the existence of four different regimes: linear, sublinear, prime, and asymptotic. We highlight that the prime regime, where the timescale of convergence to steady state achieves its smallest value, occurs for intermediate values of coupling, that is, not extreme values of discharge, where the redistribution of sediment and nutrients is the fastest across the delta top, enhancing the overall system aggradation and nourishment.

The application of this framework to specific systems in a more detailed manner opens up interesting research questions such as (1) what is the return period of the discharge that corresponds to the optimal coupling (1-year event, 10-year event, etc.) and how does it affect the evolution of those systems and their resilience to extreme events, (2) what specific locations of a delta might amplify across-process connectivity critically affecting the overall system transport timescales; and (3) how is the system transport timescale dependent on including more or less refined specification of across-process connectivity? For instance, by accounting for vegetation, topography, etc., more layers can be included, with islands of similar characteristics (i.e., islands that can be modeled by a similar diffusion coefficient) grouped in the same layer. Finally, we want to emphasize the broad applicability of this framework to diverse fields in the geosciences where multiprocess multiscale interactions dictate the overall system behavior. Examples include flux transport taking into account surface-subsurface exchange (Sawyer et al., 2015), integrated wetland and river systems (Hansen et al., 2018), interaction types among species in ecological systems (Pilosof et al., 2017), and climate networks (Donges et al., 2011).

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